

Factoring

What is it? - Finding what was multiplied together (*“the factors”*) to get an expression (*the polynomial*).

Why is it done? – To aid in solving equations.

How is it done? – Follow the general steps outlined below in order.

STEP 1: Place the terms of the polynomial in descending order. (Alphabetical by power)

Example 1 $-14x^2 + 4x^4 + 6$ \rightarrow $4x^4 - 14x^2 + 6$

Example 2 $-3x^3 + x^5 - 2x^4 - 6x^2$ \rightarrow $x^5 - 2x^4 - 3x^3 - 6x^2$

Example 3 $4q^2 + 3p^2 - 2pq$ \rightarrow $3p^2 - 2pq + 4q^2$

STEP 2: Look for common terms (**Greatest Common Factor**) that can be **“pulled”** out.
(Note: Always pull out a leading negative.)

Example 1

$$3x^3 + 9x^2 - 6x = 3x(x^2 + 3x - 2)$$

Example 2

$$-2x^4 + 12x^3 - 10x^2 = -2x^2(x^2 - 6x + 5)$$

STEP 3: Identify which type of polynomial it is (*Binomial, Trinomial, or Four-Term*)

A. **Binomial (2 terms)** – Look for one of the following special cases

i. **Difference of Squares:** $a^2 - b^2 = (a - b)(a + b)$

ii. **Sum of Squares:** $a^2 + b^2 \rightarrow$ Does not factor (Prime)

iii. **Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

iv. **Sum of Cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

If none of these occur, the Binomial *does not factor* (i.e. Prime).

Example 1 (Diff of Squares)

$$\begin{aligned} 4x^2 - 9 &= (2x)^2 - (3)^2 \\ &= (2x - 3)(2x + 3) \end{aligned}$$

Example 2 (Sum of Squares)

$$\begin{aligned} 25y^2 + 49 &= (5y)^2 + (7)^2 \\ &= \text{Does NOT Factor} \end{aligned}$$

Example 3 (Diff of Cubes)

$$\begin{aligned} 27t^3 - 8 &= (3t)^3 - (2)^3 \\ &= (3t - 2)[(3t)^2 + (3t)(2) + (2)^2] \\ &= (3t - 2)(9t^2 + 6t + 4) \end{aligned}$$

Example 4 (Sum of Cubes)

$$\begin{aligned} 64w^3 + 125z^3 &= (4w)^3 + (5z)^3 \\ &= (4w + 5z)[(4w)^2 - (4w)(5z) + (5z)^2] \\ &= (4w + 5z)(16w^2 - 20wz + 25z^2) \end{aligned}$$

Note: On Sum and Difference of Cubes, the resulting trinomial does NOT factor.

B. **Trinomial (3 terms)** - Determine whether it is a perfect square trinomial or a general trinomial.

i. **Perfect Square Trinomial** (2 cases):

a. $a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$ **OR**

b. $a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$

Example 1

$$\begin{aligned}x^2 + 8x + 16 &= (x)^2 + 2(x)(4) + (4)^2 \\ &= (x + 4)^2\end{aligned}$$

Example 2

$$\begin{aligned}4y^2 - 28y + 49 &= (2y)^2 - 2(2y)(7) + (7)^2 \\ &= (2y - 7)^2\end{aligned}$$

ii. **General Trinomial:** $ax^2 + bx + c$

a. Factor by using the *AC Method* (grouping)

b. Factor by *Trial & Error* OR *Bottoms Up* Methods

c. Factor by using the *Quadratic Formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

d. Factor by using the *Complete the Square* Method

C. **Four terms** - Attempt to factor first by using Grouping or second by using Synthetic Division if Grouping does not work.

i. **Grouping**

a. Group the first two terms together and the last two terms together.

$$x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3) = (x^2 - 4)(x - 3) = (x + 2)(x - 2)(x - 3)$$

b. Group the first three terms together and leave the last separate.

$$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2 = (x + 3)^2 - y^2 = [(x + 3) - y][(x + 3) + y] = (x - y + 3)(x + y + 3)$$

c. Group the last three terms together and leave the first separate.

$$x^2 - y^2 + 8y - 16 = x^2 - (y^2 - 8y + 16) = x^2 - (y - 4)^2 = [x - (y - 4)][x + (y - 4)] = (x - y + 4)(x + y - 4)$$

ii. **Synthetic Division:** Use the Rational Zero Test to identify the possible rational zeros and begin testing using synthetic division. The list can be narrowed down if desired by applying Descartes rule of signs, Upper and Lower bound rules, or the Intermediate Value Theorem. (*This is a College Algebra level factoring skill*)

STEP 4: Check answers by multiplying the factors back together to confirm that you have the original polynomial.

Sample Factoring Problems

<u>Problems</u>	<u>Answers</u>	<u>Steps/Methods Used</u>
1. $4s^2 + 2s^4$	$2s^2(s^2 + 2)$	Desc Order, CGF, Bin: Prime
2. $10x^2y^2 + 5x^2y - 15xy^2$	$5xy(2xy + x - 3y)$	GCF
3. $3r^2 + 4rs - 5$	Prime	No GCF
4. $x^2 - 64$	$(x - 8)(x + 8)$	Bin: Diff of Sqrs
5. $4y^2 - 49$	$(2y - 7)(2y + 7)$	Bin: Diff of Sqrs
6. $7x^2 + 28$	$7(x^2 + 4)$	GCF, Bin: Sum of Sqrs
7. $-6n^2 + 54m^2$	$6(3m - n)(3m + n)$	Dsc Or, GCF, Bin: Diff of Sqrs
8. $-3x^2 - 75$	$-3(x^2 + 25)$	GCF, Bin: Sum of Sqrs
9. $-2w^4 + 32$	$-2(w^2 + 4)(w - 2)(w + 2)$	GCF, Diff of Sqrs (<i>twice</i>)
10. $p^3 + 1$	$(p + 1)(p^2 - p + 1)$	Bin: Sum of Cubes
11. $x^3 - 8$	$(x - 2)(x^2 + 2x + 4)$	Bin: Diff of Cubes
12. $27x^3 + 125$	$(3x + 5)(9x^2 - 15x + 25)$	Bin: Sum of Cubes
13. $64 - 343t^3$	$-(7t - 4)(49t^2 + 28t + 16)$	Dsc Or, GCF, Bin: Diff of Cub
14. $\frac{1}{4}x^2 - \frac{1}{9}y^2$	$\left(\frac{1}{2}x - \frac{1}{3}y\right)\left(\frac{1}{2}x + \frac{1}{3}y\right)$	Bin: Diff of Sqrs
15. $x^4 - (y + 1)^2$	$(x^2 - y - 1)(x^2 + y + 1)$	Bin: Diff of Sqrs
16. $-4z^4 + 256z$	$-4z(z - 4)(z^2 + 4z + 16)$	GCF, Bin: Diff of Cubes
17. $x^2 + 6x + 9$	$(x + 3)^2$ OR $(x + 3)(x + 3)$	Trin: Perfect Square
18. $25 - 20y + 4y^2$	$(2y - 5)^2$ OR $(2y - 5)(2y - 5)$	Desc Order, Trin: Perf Sqr
19. $x^2 - 9x + 20$	$(x - 4)(x - 5)$	Trin: AC Method
20. $y^2 + 8y + 15$	$(y + 3)(y + 5)$	Trin: AC Method

21.	$6x^2 + 19x - 7$	$(2x + 7)(3x - 1)$	Trin: AC Method
22.	$8 - 4t^2 + 4t$	$-4(t + 1)(t - 2)$	Desc Ord, GCF, Trin: AC
23.	$3w^4 + 6w^3 + 3w^2$	$3w^2(w + 1)^2$	GCF, Trin: Perf Sqr
24.	$2r^2 - 3rs - 2s^2$	$(2r + s)(r - 2s)$	Trin: AC Method
25.	$5x^2y^2 + 25x^2y + 20x^2$	$5x^2(y + 4)(y + 1)$	GCF, Trin: AC Method
26.	$(x^2 + 2x + 1) - (y^2 + 4y + 4)$	$(x + y + 3)(x - y - 1)$	Trin: Perf Square (<i>twice</i>) Bin: Diff of Sqrs
27.	$x^4 - 2x^2 - 24$	$(x^2 + 4)(x^2 - 6)$	Trin: AC Method "Quadratic Like"
28.	$x^3 + 2x^2 - 4x - 8$	$(x - 2)(x + 2)^2$	4 Term: Grouping
29.	$4 - 3q^2 + 3q^3 - 4q$	$(3q^2 - 4)(q - 1)$	Desc Order, 4 Term Group
30.	$4m^2n^2 - 25m^2 - 16n^2 + 100$	$(m - 2)(m + 2)(2n - 5)(2n + 5)$	4 Term: Grouping Bin: Diff of Sqrs
31.	$x^2 - (y^2 + 2y + 1)$	$(x + y + 1)(x - y - 1)$	4 Term: Grp last 3 Trin: Perf Square Bin: Diff of Sqrs
32.	$64 - 4x^2 - 12xy - 9y^2$	$-(2x + 3y - 8)(2x + 3y + 8)$	Descending Order GCF (-1) 4 Term: Grp first 3 Trin: Perf Square Bin: Diff of Sqrs

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33.	$x^3 - 6x^2 + 11x - 6$	$(x - 1)(x - 2)(x - 3)$	4 Term: Synthetic Div
34.	$x^3 + 7x^2 + 14x + 8$	$(x + 1)(x + 2)(x + 4)$	4 Term: Synthetic Div
35.	$2x^4 - 4x^3 - 42x^2 - 36x$	$2x(x + 1)(x + 3)(x - 6)$	4 Term: GCF, Synthetic Div
36.	$4x^2 + 6x + x^3 - x^4$	$-x(x - 3)(x^2 + 2x + 2)$	4 Term: Desc Order, GCF, Synthetic Div