

FACTORIZING POLYNOMIALS

- 1) First determine if a common monomial factor (Greatest Common Factor) exists. Factor trees may be used to find the GCF of difficult numbers. Be aware of opposites: Ex. $(a-b)$ and $(b-a)$ These may become the same by factoring -1 from one of them.

$$\begin{aligned}3x - 12 &= 3(x - 4) \\x^2y^2 - 3xy^2 &= xy^2(x - 3) \\6(x - y) + a(x - y) &= (x - y)(6 + a)\end{aligned}$$

- 2) If the problem to be factored is a binomial, see if it fits one of the following situations.

- A. Difference of two squares:

$$\begin{aligned}a^2 - b^2 &= (a + b)(a - b) \\9x^2 - 25y^2 &= (3x + 5y)(3x - 5y) \\(a + b)^2 - 25 &= [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5)\end{aligned}$$

- B. Sum of two squares:

$$a^2 + b^2 \text{ does not factor (it is prime).}$$

- C. Sum of two cubes:

$$\begin{aligned}a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\8x^3 + 27y^3 &= (2x + 3y)(4x^2 - 6xy + 9y^2)\end{aligned}$$

Note: Resulting trinomial does not factor.

- D. Difference of two cubes:

$$\begin{aligned}a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\x^3 - 64 &= (x - 4)(x^2 + 4x + 16)\end{aligned}$$

Note: Resulting trinomial does not factor.

- E. If none of these occur, the binomial does not factor.

- 3) If the problem is a trinomial, check for one of the following possibilities.

- A. Square of a binomial:

$$\begin{aligned}a^2 + 2ab + b^2 &= (a + b)(a + b) = (a + b)^2 \\x^2 + 6x + 9 &= (x + 3)(x + 3) = (x + 3)^2 \\4x^2 - 20xy + 25y^2 &= (2x - 5y)^2\end{aligned}$$

- B. If $a = 1$, use reverse foil or trial and error method:

$$\begin{aligned}x^2 + 7x + 12 &= (x + 3)(x + 4) \\x^2 - 7x + 12 &= (x - 3)(x - 4) \\x^2 + 3x - 18 &= (x + 6)(x - 3) \\x^2 - 3x - 18 &= (x - 6)(x + 3)\end{aligned}$$

- C. If $a \neq 1$, use trial and error method. (Grouping may also be used.)

- 4) If factoring a polynomial with four terms, possible choices are below.

- A. Group first two terms together and last two terms together.

$$\begin{aligned}5a - 5b - xa + xb &= (5a - 5b) + (-xa + xb) = 5(a - b) - x(a - b) = (a - b)(5 - x) \\x^3 - 3x^2 + 2x - 6 &= (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)\end{aligned}$$

- B. Group first three terms together.

$$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2 = (x + 3)^2 - y^2 = [(x + 3) + y][(x + 3) - y] = (x + 3 + y)(x + 3 - y)$$

- C. Group last three terms together.

$$y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9) = y^2 - (x - 3)^2 = [y + (x - 3)][y - (x - 3)] = (y + x - 3)(y - x + 3)$$

BE SURE YOUR ANSWERS WILL NOT FACTOR FURTHER!

All answers may be checked by multiplication.

PRACTICE PROBLEMS:

- $y^3 + 9y^2$
- $5x^2y^3 + 15x^3y^2$
- $12t^5 - 20t^4 + 8t^2 - 16$
- $p^2 - 36$
- $25 - x^2$
- $4a^3 - 49a$
- $(a + b)^2 - 100$
- $9 - (x - y)^2$
- $y^3 + 8$
- $64y^4 + y$
- $x^3 - 27$
- $5x^3 - 40y^3$
- $2y^4 - 128y$
- $t^6 - 64$
- $x^2 - 10x + 25$
- $4a^2 + 16a + 16$
- $16y^2 + 56y + 49$
- $-20xy + 4y^2 + 25x^2$
- $x^2 + 9x + 20$
- $2y^2 - 16y + 32$
- $3x + x^2 - 10$
- $y^2 + 5y - 84$
- $8x^2 - 16 - 28x$
- $12x^3 - 31x^2 + 20x$
- $6a^2 - 7a - 10$
- $8 - 6x - 9x^2$
- $6x^6 + x^3 - 2$
- $2x^8 - 14x^4 + 20$
- $y^3 - y^2 + 2y - 2$
- $x^4 - x^3 - x + x^2$
- $x^3 + 8x^2 - x - 8$
- $p^2q - 25q + 3p^2 - 75$
- $16 - x^2 + 2xy - y^2$
- $2xy - x^2y - 6 + 3x$
- $6x^2 + 23x + 20$
- $9x^2 + 15x + 4$
- $8m^2 - 6m - 9$
- $25 - 10x + x^2$
- $16 - w^4$
- $ay - yx - x^2 + ax$

ANSWERS:

- $y^2(y + 9)$
- $5x^2y^2(y + 3x)$
- $4(3t^5 - 5t^4 + 2t^2 - 4)$
- $(p + 6)(p - 6)$
- $(5 + x)(5 - x)$
- $a(2a + 7)(2a - 7)$
- $(a + b + 10)(a + b - 10)$
- $(3 + x - y)(3 - x + y)$
- $(y + 2)(y^2 - 2y + 4)$
- $y(4y + 1)(16y^2 - 4y + 1)$
- $(x - 3)(x^2 + 3x + 9)$
- $5(x - 2y)(x^2 + 2xy + 4y^2)$
- $2y(y - 4)(y^2 + 4y + 16)$
- $(t + 2)(t^2 - 2t + 4)(t - 2)(t^2 + 2t + 4)$
- $(x - 5)^2$
- $4(a + 2)^2$
- $(4y + 7)^2$
- $(5x - 2y)^2$
- $(x + 5)(x + 4)$
- $2(y - 4)^2$
- $(x + 5)(x - 2)$
- $(y + 12)(y - 7)$
- $4(2x + 1)(x - 4)$
- $x(4x - 5)(3x - 4)$
- $(a - 2)(6a + 5)$
- $(4 + 3x)(2 - 3x)$
- $(3x^3 + 2)(2x^3 - 1)$
- $2(x^4 - 5)(x^4 - 2)$
- $(y - 1)(y^2 + 2)$
- $x(x^2 + 1)(x - 1)$
- $(x + 8)(x + 1)(x - 1)$
- $(q + 3)(p + 5)(p - 5)$
- $(4 + x - y)(4 - x + y)$
- $(2 - x)(xy - 3)$
- $(3x + 4)(2x + 5)$
- $(3x + 1)(3x + 4)$
- $(4m + 3)(2m - 3)$
- $(5 - x)^2$ or $(x - 5)^2$
- $(4 + w^2)(2 + w)(2 - w)$
- $(y + x)(a - x)$

MORE PRACTICE PROBLEMS:

41. $x^2 - 6x - 16$

42. $x^2 - 10xy + 24y^2$

43. $x^2 + 3x + 2$

44. $x^2 - 3x + 2$

45. $x^2 - x - 30$

46. $x^2 + 7x - 8$

47. $x^2 + x - 2$

48. $x^2 - 5xy + 6y^2$

49. $x^2 + 10x + 16$

50. $x^2 + x - 72$

51. $x^2 - 8x - 9$

52. $x^2 + 2x - 48$

53. $x^2 - 13xy + 42y^2$

54. $x^2 + 8x + 12$

55. $4x^3 - 8x^2 - 12x$

56. $2x^3 - 2x^2 - 4x$

57. $2x^3 - 4x^2 - 6x$

58. $3x^3 - 6x^2 - 9x$

59. $5x^3y - 35x^2y + 50xy$

60. $3x^3y + 18x^2y - 21xy$

61. $4x^2 + 1 - 4x$

62. $15x^2 + 12 + 29x$

63. $8r^2 - 2r - 3$

64. $35a^2 + 3a - 20$

65. $25x^2 + 8 + 30x$

66. $12x^2 + 3 + 13x$

67. $9x^2 - 27xy + 20y^2$

68. $25u^2 - 15u - 18$

69. $12f^2 - 4f - 5$

70. $5z^2 + 3z + 4$

71. $4x^2 + 15 + 16x$

72. $20x^2 + 6 + 23x$

73. $6x^2 - 19xy + 10y^2$

74. $35p^2 + 13p - 4$

75. $50x^2 + 10x - 12$

76. $-30x^2 - 25x + 30$

77. $-18x^2 + 18x + 20$

78. $3x^3 - 22x^2 + 7x$

79. $15x^2 - 18x - 24$

80. $4x^3 - 25x^2 + 6x$

ANSWERS:

41. $(x - 8)(x + 2)$ 42. $(x - 6y)(x - 4y)$ 43. $(x + 2)(x + 1)$ 44. $(x - 2)(x - 1)$

45. $(x - 6)(x + 5)$ 46. $(x + 8)(x - 1)$ 47. $(x + 2)(x - 1)$ 48. $(x - 3y)(x - 2y)$

49. $(x + 8)(x + 2)$ 50. $(x + 9)(x - 8)$ 51. $(x - 9)(x + 1)$ 52. $(x + 8)(x - 6)$

53. $(x - 7y)(x - 6y)$ 54. $(x + 6)(x + 2)$ 55. $4x(x - 3)(x + 1)$ 56. $2x(x - 2)(x + 1)$

57. $2x(x - 3)(x + 1)$ 58. $3x(x - 3)(x + 1)$ 59. $5xy(x - 5)(x - 2)$ 60. $3xy(x + 7)(x - 1)$ 61.

$(2x - 1)^2$ 62. $(3x + 4)(5x + 3)$ 63. $(2r + 1)(4r - 3)$ 64. $(5a + 4)(7a - 5)$

65. $(5x + 4)(5x + 2)$ 66. $(3x + 1)(4x + 3)$ 67. $(3x - 5y)(3x - 4y)$ 68. $(5u + 3)(5u - 6)$ 69.

$(2f + 1)(6f - 5)$ 70. *Prime (Cannot be factored)* 71. $(2x + 3)(2x + 5)$

72. $(5x + 2)(4x + 3)$ 73. $(2x - 5y)(3x - 2y)$ 74. $(7p + 4)(5p - 1)$

75. $2(5x + 3)(5x - 2)$ 76. $-5(2x + 3)(3x - 2)$ 77. $-2(3x - 5)(3x + 2)$

78. $x(3x - 1)(x - 7)$ 79. $3(5x + 4)(x - 2)$ 80. $x(4x - 1)(x - 6)$

MORE PRACTICE PROBLEMS:

81. $125x^3 - 1$
82. $w^2 - 64$
83. $y^2 - 12y + 36$
84. $x^2 - 8x - 48$
85. $a^3 - 7a^2 + 12a$
86. $25a^2 + 8b^2$
87. $(x - 3)(x + 7) + (x - 3)(x - 4)$
88. $6x^2 + 12x + 6$
89. $y^2 - 11y + 18$
90. $40 + 3b - b^2$
91. $3x^5 - 12x^2$
92. $250x^3 + 2$
93. $7xy^4 - 7xz^4$
94. $2y^4 + 5y^3 - 12y^2$
95. $24x^2 - 7x - 5$
96. $y^2 + 14y - 32$
97. $0.04w^2 + 0.28w + 0.49$
98. $4x^3 + 40x^2 + 64x$
99. $64y^3 + 27$
100. $\frac{1}{81} - x^2$
101. $5x^2 - 2x + 3$
102. $x^3 - 343$
103. $40y^2 + 28y - 48$
104. $3ab - 5bc + bd$
105. $8c^6 - 125d^6$
106. $81 - 18z + z^2$
107. $x^4 + 10x^3 + 25x^2$
108. $xz - xw - yz + yw$
109. $y^2 + 5y - 36$
110. $x^2 - 11x - 42$
111. $7a^2 - 7b^2$
112. $216 - a^3$
113. $81 + 18y + y^2$
114. $b^2 - 5b - 14$
115. $q^4 - 10q^3 + 21q^2$
116. $9x^2y^2 - 25y^4$
117. $105 + 8x - x^2$
118. $x^2 - 3x - 2$
119. $6y^3 + 48$
120. $a^3 - 14a^2 + 49a$

ANSWERS:

81. $(5x - 1)(25x^2 + 5x + 1)$ 82. $(w + 8)(w - 8)$ 83. $(y - 6)^2$ 84. $(x - 12)(x + 4)$
85. $a(a - 4)(a - 3)$ 86. *Prime (Cannot be factored)* 87. $(x - 3)(2x + 3)$
88. $6(x + 1)^2$ 89. $(y - 9)(y - 2)$ 90. $(8 - b)(5 + b)$ 91. $3x^2(x^3 - 4)$
92. $2(5x + 1)(25x^2 - 5x + 1)$ 93. $7x(y^2 + z^2)(y + z)(y - z)$ 94. $y^2(2y - 3)(y + 4)$
95. $(8x - 5)(3x + 1)$ 96. $(y - 2)(y + 16)$ 97. $(0.2w + 0.7)^2$ 98. $4x(x + 2)(x + 8)$
99. $(4y + 3)(16y^2 - 12y + 9)$ 100. $\left(\frac{1}{9} + x\right)\left(\frac{1}{9} - x\right)$ 101. *Prime (Cannot be factored)* 102.
 $(x - 7)(x^2 + 7x + 49)$ 103. $4(2y + 3)(5y - 4)$ 104. $b(3a - 5c + d)$
105. $(2c^2 - 5d^2)(4c^4 + 10c^2d^2 + 25d^4)$ 106. $(9 - z)^2$ 107. $x^2(x + 5)^2$
108. $(x - y)(z - w)$ 109. $(y - 4)(y + 9)$ 110. $(x - 14)(x + 3)$ 111. $7(a + b)(a - b)$
112. $(6 - a)(36 + 6a + a^2)$ 113. $(9 + y)^2$ 114. $(b - 7)(b + 2)$ 115. $q^2(q - 3)(q - 7)$ 116.
 $y^2(3x + 5y)(3x - 5y)$ 117. $(7 + x)(15 - x)$ 118. *Prime (Cannot be factored)*
119. $6(y + 2)(y^2 - 2y + 4)$ 120. $a(a - 7)^2$

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| 121. | $3y^2 - 34y - 24$ | 131. | $x^2 - 0.6x + 0.09$ |
| 122. | $a^2 + 8a + 16$ | 132. | $4x^2 - 13x - 35$ |
| 123. | $y^2 - 121$ | 133. | $125x^6 - 81$ |
| 124. | $42 + a - a^2$ | 134. | $49x^3 - 14x^2 + x$ |
| 125. | $9x^3 - 24x^2 + 16x$ | 135. | $40y^2 + 7y - 3$ |
| 126. | $x^3 - \frac{1}{8}$ | 136. | $15w^2 - 15w - 90$ |
| 127. | $10w^2 + 29w - 21$ | 137. | $0.04a^2 - 0.49b^2$ |
| 128. | $16x^2 + 54x - 7$ | 138. | $x^3y^2 + 7x^2y^2 - 18xy^2$ |
| 129. | $27x^2 - 30x - 8$ | 139. | $2x^6 - 54y^6$ |
| 130. | $x^6 - 1$ | 140. | $\frac{1}{4}x^2 - 5x + 25$ |

ANSWERS:

121. $(y - 12)(3y + 2)$ 122. $(a + 4)^2$
 123. $(y + 11)(y - 11)$ 124. $(7 - a)(6 + a)$ 125. $x(3x - 4)^2$ 126. $\left(x - \frac{1}{2}\right)\left(x^2 + \frac{1}{2}x + \frac{1}{4}\right)$
 127. $(5w - 3)(2w + 7)$ 128. $(2x + 7)(8x - 1)$ 129. $(9x + 2)(3x - 4)$
 130. $(x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$ 131. $(x - 0.3)^2$ 132. $(x - 5)(4x + 7)$
 133. *Prime (Cannot be factored)* 134. $x(7x - 1)^2$ 135. $(8y + 3)(5y - 1)$
 136. $15(w + 2)(w - 3)$ 137. $(0.2a + 0.7b)(0.2a - 0.7b)$ 138. $xy^2(x - 2)(x + 9)$
 139. $2(x^2 - 3y^2)(x^4 + 3x^2y^2 + 9y^4)$ 140. $\left(\frac{1}{2}x - 5\right)^2$