Math 130 – Trigonometry
Final Exam Review (revised Fall 2023)

Part I. Multiple Choice: Choose the best possible answer.

1. Given $\cos u = -\frac{2}{7}$ with u in Quadrant II, find $\cos\frac{u}{2}$ and $\sin 2u$.

   a. $\cos\frac{u}{2} = -\frac{\sqrt{70}}{14}$ $\quad \sin 2u = \frac{6\sqrt{5}}{7}$

   b. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{6\sqrt{5}}{7}$

   c. $\cos\frac{u}{2} = \frac{3\sqrt{14}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

   d. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$ $\quad \sin 2u = -\frac{12\sqrt{5}}{49}$

For problems 2 and 3, let $\sin A = -\frac{7}{25}$ with A in Quadrant III and $\cos B = -\frac{4}{5}$ with B in Quadrant III.

2. Find $\sin(A + B)$

   a. $-\frac{4}{5}$ $\quad$ b. $\frac{3}{5}$ $\quad$ c. $\frac{4}{5}$ $\quad$ d. $-\frac{3}{5}$

3. Find $\tan(A - B)$

   a. $\frac{100}{117}$ $\quad$ b. $-\frac{44}{75}$ $\quad$ c. $\frac{44}{75}$ $\quad$ d. $-\frac{44}{117}$

4. Simplify the trigonometric expression: $\frac{\sec\theta - 1}{1 - \cos\theta}$

   a. $\sec\theta$ $\quad$ b. $\cos\theta$ $\quad$ c. $\frac{\sec\theta + \cos\theta}{\sin^2\theta}$ $\quad$ d. -1

5. Simplify the trigonometric expression: $\frac{1}{\cos x + 1} + \frac{1}{\cos x - 1}$

   a. $\sec x$ $\quad$ b. $-2\csc x \cot x$ $\quad$ c. $-2\csc^2 x$ $\quad$ d. $\frac{2}{\cos^2 x - 1}$

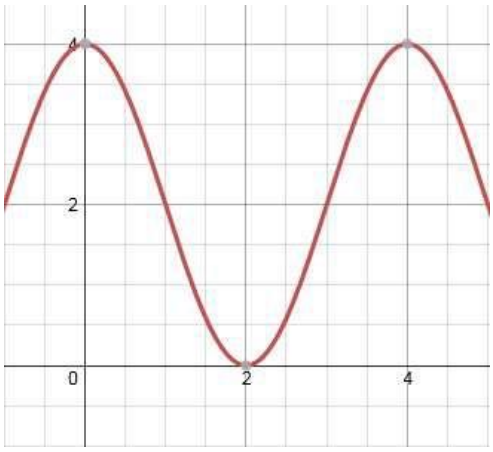
6. Given $\sin\theta = -\frac{12}{15}$ and $tan\theta < 0$, find the five remaining trigonometric functions of θ.

a.	b.	c.	d.
$\csc\theta = -\frac{15}{12}$	$\csc\theta = \frac{15}{12}$	$\csc\theta = -\frac{15}{12}$	$\csc\theta = \frac{15}{12}$
$\cos\theta = \frac{9}{15}$	$\cos\theta = -\frac{9}{15}$	$\cos\theta = -\frac{9}{15}$	$\cos\theta = \frac{9}{15}$
$\sec\theta = \frac{15}{9}$	$\sec\theta = -\frac{15}{9}$	$\sec\theta = \frac{-15}{9}$	$\sec\theta = \frac{15}{9}$
$\tan\theta = -\frac{12}{9}$	$\tan\theta = -\frac{12}{9}$	$\tan\theta = \frac{12}{9}$	$\tan\theta = \frac{12}{9}$
$\cot\theta = -\frac{9}{12}$	$\cot\theta = -\frac{9}{12}$	$\cot\theta = \frac{9}{12}$	$\cot\theta = \frac{9}{12}$

7. Identify the amplitude, period, horizontal shift and vertical shift for the following function: $f(x) = 1 - 3\sin(2x + \pi)$.

a.	b.	c.	d.
Amp = 3	Amp = 3	Amp = -3	Amp = 3
Per = 2π	Per = π	Per = π	Per = π
HS = left $\frac{\pi}{2}$	HS = right $\frac{\pi}{2}$	HS = left $\frac{\pi}{2}$	HS = left $\frac{\pi}{2}$
VS = up 1	VS = up 1	VS = down 1	VS = up 1

8. Find the equation that matches the graph:



a.	$y = 2 + 2\cos\left(\frac{\pi x}{2}\right)$	b.	$y = 2 + 2\cos(2\pi x)$
c.	$y = 4 + \cos\left(\frac{\pi x}{2}\right)$	d.	$y = 2 + 2\cos\left(\frac{x}{2}\right)$

9. Evaluate $\tan^{-1}(-1)$

a. $\frac{3\pi}{4}, \frac{7\pi}{4}$ b. $\frac{7\pi}{4}$ c. $-\frac{\pi}{4}$ d. $\frac{3\pi}{4}$

10. Evaluate $\cos^{-1}(-\frac{\sqrt{2}}{2})$

a. $\frac{\pi}{4}, \frac{7\pi}{4}$ b. $-\frac{\pi}{4}$ c. $\frac{3\pi}{4}$ d. $\frac{\pi}{4}$

11. Evaluate $sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}\,\pi}{2}$

12. Evaluate $sec\left(\tan^{-1}\left(\frac{1}{2x}\right)\right)$

a. $\sqrt{4x^2+1}$ b. $\frac{\sqrt{4x^2+1}}{2x}$ c. $\frac{\sqrt{2x^2+1}}{2x}$ d. $\frac{2x}{\sqrt{4x^2+1}}$

13. In triangle ABC, if $a = 3.7\ cm, c = 6.4\ cm, and\ B = 23°,$ find $b.$

a. 4.1 cm b. 3.3 cm c. 5.7 cm d. 11.1 cm

14. In triangle ABC, if $B = 110°$, $C = 40°$, and $b = 18.0$ inches, find a.

a. 9.6 inches b. 12.3 inches c. 33.8 inches d. Not enough information

15. In triangle ABC, if $a = 4.8\ in, b = 6.3\ in, and\ c = 7.5\ in,$ find the area of the triangle.

a. 4.9 in^2 b. 15.0 in^2 c. 45.9 in^2 d. 18.0 in^2

16. In triangle ABC, if $B = 57°$, $a = 7.3$ cm, $c = 3.8$ cm, find the area of the triangle.

a. 43.4 m^2 b. 23.0 m^2 c. 46.0 m^2 d. 11.6 m^2

17. Give an angle between 0° and 360° coterminal with the angle 475°:

a. 15° b. 215° c. 115° d. 205°

18. Given central angle $\theta = \frac{3\pi}{4}$ and radius 4 inches, find the arc length s.

a. $\frac{3\pi}{16}$ inches b. 12π inches c. 3π inches d. 6π inches

19. Given central angle $\theta = 72°$ and radius 5m, find the area of the sector of the circle.

a. 5π m^2 b. 10π m^2 c. 900π m^2 d. $\frac{\pi}{5}$ m^2

20. Convert the complex number $2 - 2\sqrt{3}\,i$ to trigonometric form:

a. 4cis60° b. 4cis330° c. 4cis300° d. 4cis30°

21. Multiply: 6cis120° • 3cis40°

a. 18cis80° b. 18cis160° c. 9cis160° d. 2cis80°

22. Divide: $\frac{15cis225°}{5cos45°}$

a. 3cis270° b. 5cis180° c. 75cis270° d. 3cis180°

23. Use deMoivre's Theorem to evaluate $\left(\sqrt{3}cis110°\right)^6$

a. $6\sqrt{3}cis660°$ b. $6\sqrt{3}cis300°$ c. $27cis300°$ d. $27cis660°$

24. Find two square roots of 81cis120°.

a. 3cis60°, 3cis300° b. 9cis60°, 9cis240°

c. 3cis60°, 3cis240° d. 9cis120°, 9cis300°

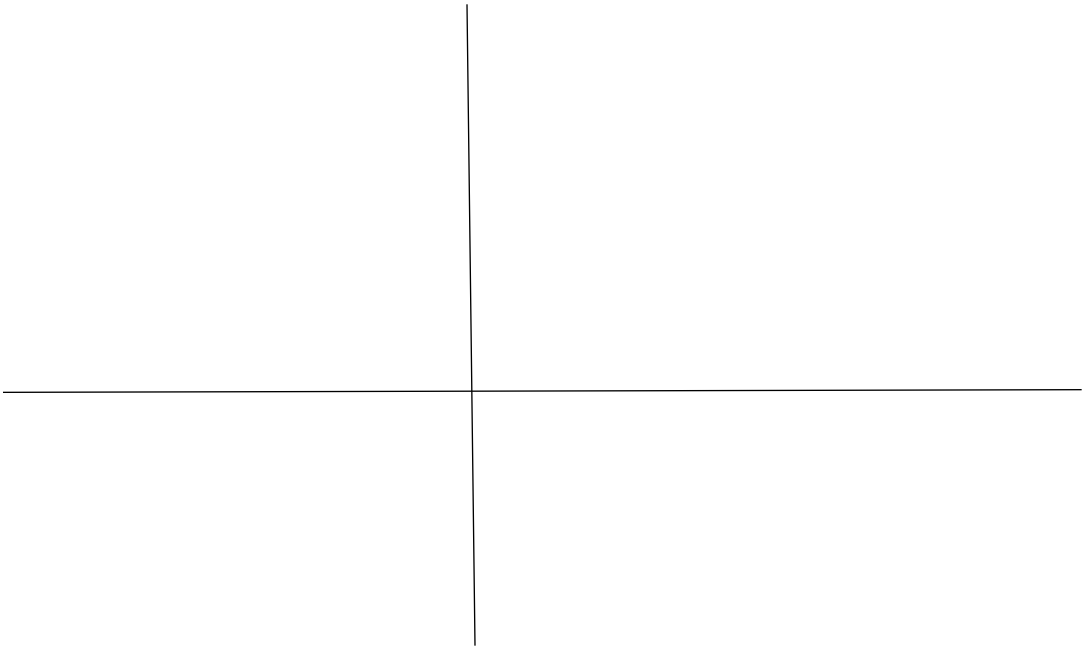
Part II. Short Answer Section: Show your work.

1. Two straight wires are strung on opposite sides of a tent pole and anchored to the ground by two stakes. One of the wires is 56 feet long and makes an angle of 47° with the ground. The other wire is 65 feet long and makes an angle of 37° with the ground. How far apart are the stakes that hold the wires to the ground?

2. Given the following trigonometric function: $y = 2 + 2\sec(x - \frac{\pi}{4})$

   a. Find the period, amplitude, horizontal translation, and vertical translation.

   b. Graph (at least one period):



3. Prove the following:

a. $\cot x - \tan x = \dfrac{\cos 2x}{\sin x \cos x}$

b. $\sin\left(60° + x\right) + \sin\left(60° - x\right) = \sqrt{3}\cos x$

c. $\left(1 - \sin\theta\right)\left(1 + \sin\theta\right) = \cos^2\theta$

d. $\cot A = \dfrac{\sin 2A}{1 - \cos 2A}$

4. Solve the trigonometric equations:

a. $\csc^2 x + 3\csc x - 4 = 0$ over $[0, 2\pi)$

b. $2\sin^2 x + 5\cos x - 4 = 0$

c. $2\sin 2x + \sqrt{3} = 0$ over $[0, 2\pi)$

d. $\sec 4x - 2 = 0$

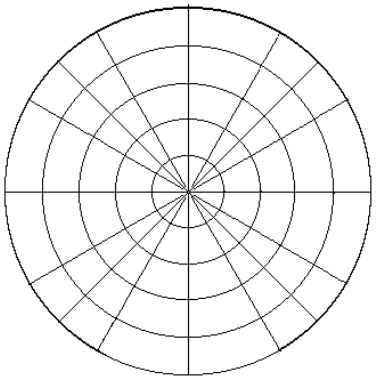
5. Given $\mathbf{u} = \langle -2, 5 \rangle$ and $\mathbf{v} = \langle -1, -8 \rangle$, find the following:

   a. The magnitude and direction angle of vector $\mathbf{u}$

   b. The magnitude and direction angle of vector $\mathbf{v}$

   c. The dot product $\mathbf{u} \cdot \mathbf{v}$

6. Plot the polar coordinates $\left(-3, \frac{5\pi}{6}\right)$ on the polar graph below. Then convert to rectangular coordinates.



7. Convert $\left(1, -\sqrt{3}\right)$ to polar coordinates. Give two possible solutions with θ is between 0 and 2π.

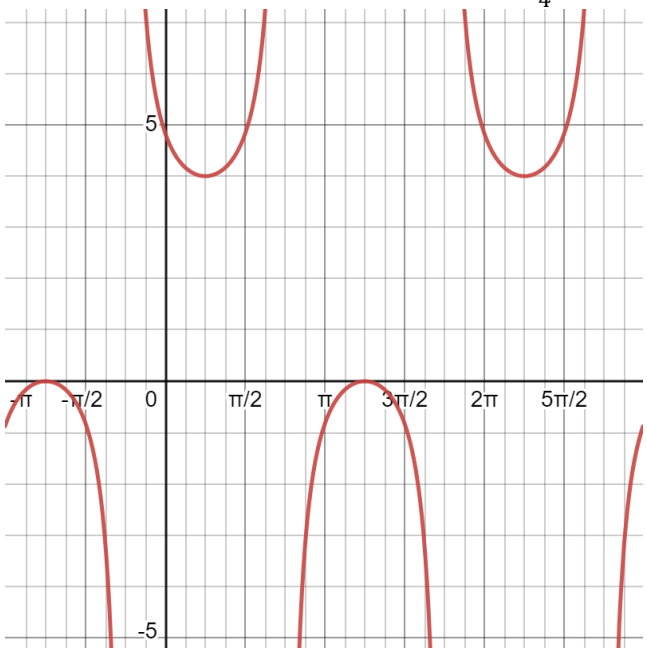
ANSWER KEY

Part I: Multiple Choice

1. D	2. C	3. D	4. A
5. B	6. A	7. D	8. A
9. C	10. C	11. A	12. B
13. B	14. A	15. B	16. D
17. C	18. C	19. A	20. C
21. B	22. D	23. D	24. B

Part II: Short Answer

1. 90.1 feet
2. Period: 2π Amp: n/a HT: right $\frac{\pi}{4}$ VT: up 2



3. a.

$$\cot x - \tan x$$
$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$
$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$
$$= \frac{\cos 2x}{\sin x \cos x}$$

b.

$$\sin(60° + x) + \sin(60° - x)$$
$$= \sin 60° \cos x + \cos 60° \sin x + \sin 60° \cos x - \cos 60° \sin x$$
$$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$
$$= \sqrt{3}\cos x$$

3. c.

$$(1-\sin\theta)(1+\sin\theta)$$
$$=1+\sin\theta-\sin\theta-\sin^2\theta$$
$$=1-\sin^2\theta$$
$$=\cos^2\theta$$

d.

$$\frac{\sin 2A}{1-\cos 2A}$$
$$=\frac{2\sin A\cos A}{1-\left(1-2\sin^2 A\right)}$$
$$=\frac{2\sin A\cos A}{2\sin^2 A}$$
$$=\frac{\cos A}{\sin A}$$
$$=\cot A$$

4. a. $x=\frac{\pi}{2}, 3.39, 6.54$

   b. $\frac{\pi}{3}+2\pi k, \frac{5\pi}{3}+2\pi k$

   c. $x=\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

   d. $\frac{\pi}{12}+\frac{\pi}{2}k, \frac{5\pi}{12}+\frac{\pi}{2}k$

5. a. $\left|\vec{u}\right|=\sqrt{29}$, $\theta = 111.8°$

   b. $\left|\vec{v}\right|=\sqrt{65}$, $\theta = 262.9°$

   c. -38

6. $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

7. $\left(2, \frac{5\pi}{3}\right), \left(-2, \frac{2\pi}{3}\right)$