Math 150 – Pre-Calculus
Final Exam Review (Nov. 2022)

Part I. Multiple Choice: Choose the best possible answer.

1. Find the domain of the function: $f(x) = \sqrt{2x+10} - 4$

   a. $(-\infty, -5] \cup [-4, \infty)$ b. $(-5, -\infty)$ c. $[-5, \infty)$ d. $[-4., \infty)$

2. Find the domain of the function: $f(x) = \frac{x+2}{x^2-5x+4}$

   a. $(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$ b. $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$

   c. $(-\infty, -2) \cup (-2, 1) \cup (1, 4) \cup (4, \infty)$ d. $(-\infty, \infty)$

3. Which of the following equations represent *y* *__as a function__* *of x*?

   a. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ b. $y = 3x^2 + 9$ c. $|y| = x - 10$ d. $x^2 + y^2 = 16$

4. Find the average rate of change of the function $f(x) = x^2 - 2x + 8$ from $x_1 = 2$ to $x_2 = 5$.

   a. 5 b. 3 c. -5 d. 11

5. Is $f(x) = x^4 - 2x^2 + 3$ even, odd, or neither? Does it have any symmetry?

   a. Odd with origin symmetry b. Even with x-axis symmetry

   c. Neither with no symmetry d. Even with y-axis symmetry

6. Find the inverse function $f^{-1}(x)$ of $f(x) = x^2 - 4$, $x \geq 0$

   a. $f^{-1}(x) = \sqrt{x-4}$ b. $f^{-1}(x) = \sqrt{x^2-4}$

   c. $f^{-1}(x) = \frac{1}{x^2-4}$ d. $f^{-1}(x) = \sqrt{x+4}$

7. Simplify and express the answer in standard form:

$$\left(-2+\sqrt{-18}\right)-\left(4+3\sqrt{2}\,i\right)$$

a. -6 b. $-6+6\sqrt{2}\,i$ c. $-6-9\sqrt{2}\,i$ d. 2

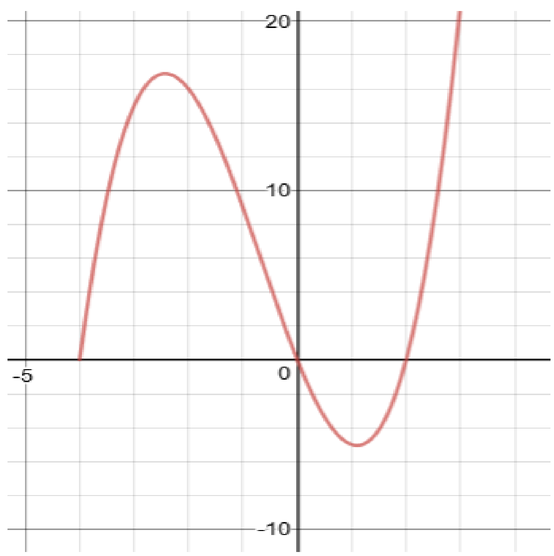
8. Multiply and express the answer in standard form: $(6+2i)(4-3i)$

a. 18 + 10i b. 18 – 10i c. 15+3i d. 30 – 10i

9. Solve the rational inequality: $\frac{x^2+x-6}{x} \geq 0$

a. $[-3\,,0] \cup [2,\infty)$ b. $(0,2]$ c. $[-3\,,0) \cup [2,\infty)$ d. $(-\infty,-3] \cup (0,2]$

10. From the graph below, find the domain and range of the function and identify the intervals where the function is decreasing.



a. Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$
Decreasing: $(0,2)$

b. Domain: $[-4,\infty)$
Range: $[-5,\infty)$
Decreasing: $(-2.5, 1.2)$

c. Domain: $[-5,\infty)$
Range: $[-4,\infty)$
Decreasing: $(0,2)$

d. Domain: $(-4,\infty)$
Range: $(-5,\infty)$
Decreasing: $(-2.5, 1.2)$

11. Which exponential equation is converted properly to logarithmic form?

a. $a = b^y \rightarrow y = log_b\ a$ b. $a = b^y \rightarrow b = log_y\ a$

c. $a = b^y \rightarrow y = log_a\ b$ d. $a = b^y \rightarrow b = log_a\ y$

12. Find the domain of the function: $f(x) = \frac{1}{2}\log(x+6) - 4$

a. $(-\infty, \infty)$ b. $[-6, 0)$ c. $(-6, \infty)$ d. $(6, \infty)$

13. Condense the logarithmic expression to a single quantity:

$$2\ log_3 x + \ log_3 y - \frac{1}{3}\ log_3 z$$

a. $log_3\left(\frac{2xy}{3z}\right)$ b. $log_3\left(\frac{x^2y}{\sqrt[3]{z}}\right)$ c. $log\left(\frac{x^2y}{\sqrt[3]{z}}\right)$ d. $log_3\sqrt[3]{\frac{x^2y}{z}}$

14. Expand the expression by using the properties of logarithms:

$$log_2\left(\frac{4m\sqrt{n}}{p^2}\right)$$

a. $log_2 4 + \ log_2 m + \frac{1}{2}log_2 n - 2log_2 p$ b. $log_2 4m + \frac{1}{2}log_2 n - 2log_2 p$

c. $log_2 4 + \ log_2 m + \frac{1}{2}log_2 n + 2log_2 p$ d. $2 + log_2 m + \frac{1}{2}log_2 n - 2log_2 p$

15. Given $\cos u = -\frac{2}{7}$ and $\frac{\pi}{2} < u < \pi$, find $\cos\frac{u}{2}$ and $\sin 2u$.

a. $\cos\frac{u}{2} = -\frac{\sqrt{70}}{14}$

$\sin 2u = \frac{6\sqrt{5}}{7}$

b. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$

$\sin 2u = -\frac{6\sqrt{5}}{7}$

c. $\cos\frac{u}{2} = \frac{3\sqrt{14}}{14}$

$\sin 2u = -\frac{12\sqrt{5}}{49}$

d. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$

$\sin 2u = -\frac{12\sqrt{5}}{49}$

For problems 16 and 17, let $\sin A = -\frac{7}{25}$ with A in Quadrant III and $\cos B = -\frac{4}{5}$ with B in Quadrant III.

16. Find $\sin(A + B)$

a. $-\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{4}{5}$ d. $-\frac{3}{5}$

17. Find $\tan(A - B)$

a. $\frac{100}{117}$ b. $-\frac{44}{75}$ c. $\frac{44}{75}$ d. $-\frac{44}{117}$

18. Simplify the trigonometric expression: $\frac{\sec\theta - 1}{1 - \cos\theta}$

a. $\sec\theta$ b. $\cos\theta$ c. $\frac{\sec\theta + \cos\theta}{\sin^2\theta}$ d. -1

19. Simplify the trigonometric expression: $\frac{1}{\cos x + 1} + \frac{1}{\cos x - 1}$

a. $\sec x$ b. $-2\csc x \cot x$ c. $-2\csc^2 x$ d. $\frac{2}{\cos^2 x - 1}$

20. Given $\sin\theta = -\frac{12}{15}$ and θ terminates in Quadrant III, find the five remaining trigonometric functions of θ.

a.	b.	c.	d.
$\csc\theta = -\frac{15}{12}$	$\csc\theta = \frac{15}{12}$	$\csc\theta = -\frac{15}{12}$	$\csc\theta = \frac{15}{12}$
$\cos\theta = \frac{9}{15}$	$\cos\theta = -\frac{9}{15}$	$\cos\theta = -\frac{9}{15}$	$\cos\theta = \frac{9}{15}$
$\sec\theta = \frac{15}{9}$	$\sec\theta = -\frac{15}{9}$	$\sec\theta = \frac{-15}{9}$	$\sec\theta = \frac{15}{9}$
$\tan\theta = -\frac{12}{9}$	$\tan\theta = -\frac{12}{9}$	$\tan\theta = \frac{12}{9}$	$\tan\theta = \frac{12}{9}$
$\cot\theta = -\frac{9}{12}$	$\cot\theta = -\frac{9}{12}$	$\cot\theta = \frac{9}{12}$	$\cot\theta = \frac{9}{12}$

21. Identify the amplitude, period, horizontal shift and vertical shift for the following function: $f(x) = 1 - 3\sin(2x + \pi)$.

a.	Amp = 3	b.	Amp = 3	c.	Amp = -3	d.	Amp = 3
	Per = 2π		Per = π		Per = π		Per = π
	HS = $-\frac{\pi}{2}$		HS = $\frac{\pi}{2}$		HS = $-\frac{\pi}{2}$		HS = $-\frac{\pi}{2}$
	VS = 1		VS = 1		VS = -1		VS = 1

22. Evaluate $\tan^{-1}(-1)$

a. $\frac{3\pi}{4}, \frac{7\pi}{4}$ b. $\frac{7\pi}{4}$ c. $-\frac{\pi}{4}$ d. $\frac{3\pi}{4}$

23. Evaluate $sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}\,\pi}{2}$

24. In triangle ABC, if $a = 3.7\ cm, c = 6.4\ cm, and\ B = 23°$, find *b.*

a. 4.1 cm b. 3.3 cm c. 5.7 cm d. 11.1 cm

25. In triangle ABC, if $a = 4.8\ in, b = 6.3\ in, and\ c = 7.5\ in,$ find the Area of the triangle.

a. 4.9 in^2 b. 15.0 in^2 c. 45.9 in^2 d. 18.0 in^2

Part II. Short Answer Section: Show your work.

1. Find the difference quotient: $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = 5x - x^2$.

2. Given the piecewise function: $f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$ find

   a. $f(-1)$ b. $f(0)$ c. $f(2)$

   d. Sketch the function:

3. Identify the parent function $f(x)$ and describe the steps (in order) that you would take to graph using transformations: $g(x) = -\sqrt{x-4} + 3$. Graph the function $g(x)$.

   a. Parent function: $f(x) =$ c. Graph $g(x)$

   b. Transformations:

4. Find the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$ using the following functions: $f(x) = \sqrt[3]{x-5}$ and $g(x) = x^3 + 1$. Are $f(x)$ and $g(x)$ inverse functions of each other? Explain why or why not.

5. Given the function. $f(x) = x^3 + 2x^2 + 4x + 8$.

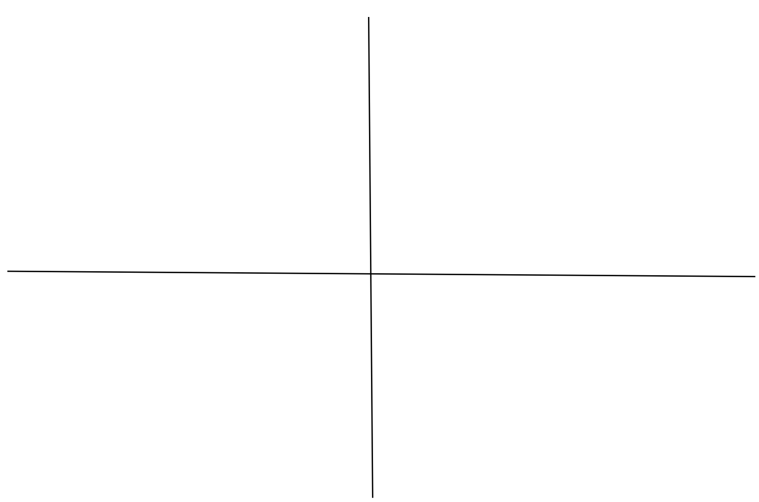
a. Factor the polynomial over the real numbers as the product of linear factors or irreducible quadratic factors.

b. State all zeros (real and imaginary) and their associated multiplicities.

c. State the end behavior.

d. Find the y-intercept.

e. Using the information in parts a – d above as a guide, sketch $f(x)$.



6. Find all asymptotes that exist (vertical, horizontal and/or slant) for the following function: $f(x) = \frac{-4x^2+1}{x^2+x-2}$. If they do not exist, explain why.

<u>Vertical</u> <u>Horizontal</u> <u>Slant</u>

7. Solve the following exponential and logarithmic equations. Leave answers in exact form:

a. $8^x = 32^{x-1}$

b. $5^x + 8 = 26$

c. $log_2 x + log_2(x+2) = log_2(x+6)$

d. $log(8x) - log(x+1) = 2$

8. Given **u** = $\langle -2, 5 \rangle$ and **v** = $\langle -1, -8 \rangle$, find the following:

a. The magnitude and direction angle of vector **u**

b. The magnitude and direction angle of vector **v**

c. The dot product **u** · **v**

9. Solve the following non-linear system: $\begin{cases} x - 2y = -6 \\ x^2 - y = 0 \end{cases}$.

Interpret your result:

Did you use the substitution method or the elimination method? Explain your reasoning.

10. Perform partial fraction decomposition on the following function:

$$f(x) = \frac{6x^2 - 5x + 19}{x^3 - x^2 + 4x - 4}$$

11. Given the following trigonometric function: $y = 2 + 2\sec(x - \frac{\pi}{4})$

a. Find the period, amplitude, horizontal translation, and vertical translation.

Amplitude:

Period:

Horizontal Translation:

Vertical Translation:

b. Graph (at least one period):

12. Solve the trigonometric equations:

a. $\csc^2 x + 3\csc x - 4 = 0$ over $[0, 2\pi)$

b. $2\sin^2 x + 5\cos x - 4 = 0$

c. $2\sin 2x + \sqrt{3} = 0$ over $[0, 2\pi)$

d. $\sec 4x - 2 = 0$

Answer Key

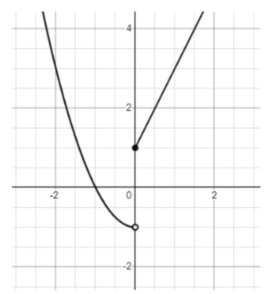
Part 1: Multiple Choice Section

1.	C	10.	B	19.	B
2.	B	11.	A	20.	C
3.	B	12.	C	21.	D
4.	A	13.	B	22.	C
5.	D	14.	D	23.	A
6.	D	15.	D	24.	B
7.	A	16.	C	25.	B
8.	D	17.	D		
9.	C	18.	A		

Part 2: Short Answer Section

1. $5 - 2x - h$

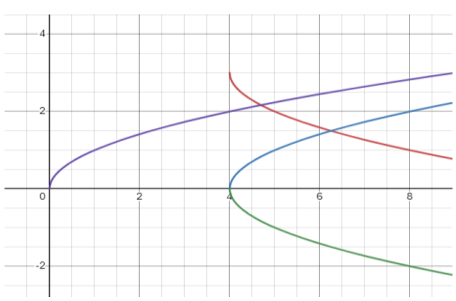
2a. 0 2b. 1 2c. 5 2d.



3a. $f(x) = \sqrt{x}$

3b. Right by 4
x-axis reflection
Up 3

3c.



Parent: Purple

Right 4: Blue

x-axis reflection: Green

Up 3: Red

4. $(f \circ g)(x) = \sqrt[3]{x^3 - 4}$, $(g \circ f)(x) = x - 4$,

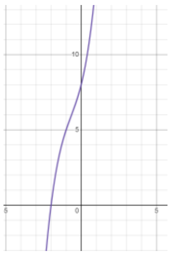
No, they are NOT inverses because $(f \circ g)(x) \neq (g \circ f)(x) \neq x$

5a. $f(x) = (x+2)(x^2+4)$

5b. Zeros: $x = -2\ (mult\ 1), x = 2i\ (mult\ 1), x = \ -2i\ (mult\ 1)$

5c. Down left, Up right

5d. $(0, 8)$

5e. 

6. VA: $x = -2,\ x = 1$

HA: $y = \ -4$

Slant: No slant asymptote. Power of numerator is not greater than power of denominator by 1.

7a. $x = \frac{5}{2}$,

7b. $x = \frac{\ln 18}{\ln 5}\ OR = \frac{\log 18}{\log 5}$ OR $= log_5 18$

7c. $x = 2$

7d. No Solution

8a. $\sqrt{29},\ \theta = \arctan(-\frac{5}{2}) = \ 111.8°$

8b. $\sqrt{65},\ \theta = \arctan(8) = 262.9°$

8c. -38

9a. $(2, 4), (-\frac{3}{2}, \frac{9}{4})$

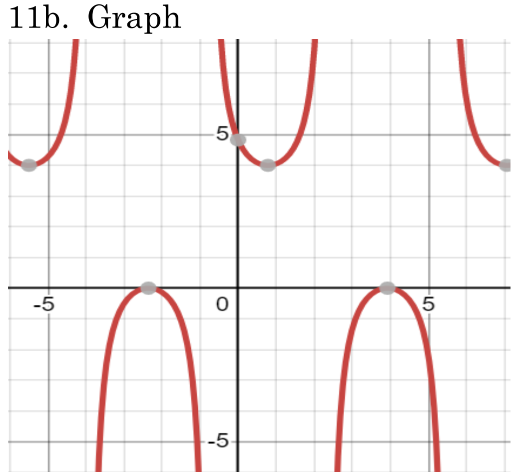
9b. The parabola and line intersect in two points

9c. Either method requires about the same amount of work.

10. $\frac{4}{x-1} + \frac{2x-3}{x^2+4}$

11a. *Amplitude:* none
Period: 2π
Horizontal Translation: $\frac{\pi}{4}$ to the right
Vertical Translation: 2 up

11b. Graph



12a. $x = \frac{\pi}{2}$, $x = \arcsin\left(\frac{1}{4}\right)$, $x = \arcsin\left(-\frac{1}{4}\right)$

12b. $x = \frac{\pi}{3} + 2n\pi$, $x = \frac{5\pi}{3} + 2n\pi$

12c. $\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

12d. $x = \frac{\pi}{12} + \frac{n\pi}{2}$. $x = \frac{5\pi}{12} + \frac{n\pi}{2}$