Math 190 (Calculus II) Final Review

1. Sketch the region enclosed by the given curves and find the area of the region.

   a. $y = 7 - 2x^2\ ,\quad y = x^2 + 4$

   b. $y = \cos\left(\frac{\pi x}{2}\right)\ ,\quad y = 1 - x^2$

2. Use the specified method to find the volume generated by rotating the region bounded by the given curves and axis of rotation.

   a. Use the Disk/Washer Method: $y = 4 - x^2,\ y = 2 - x\ ,\quad About\ x - axis$

   b. Use the Shell Method: $y = 2x - x^2\ ,\ y = x\ ,\ About\ the\ line\ x = 1$

3. a. Find the Arc Length of the graph of the function over the indicated interval.

   $$y = \frac{x^7}{14} + \frac{1}{10x^5}\ ,\quad [1\ ,2]$$

   b. Find the Surface Area generated by revolving the curve on the indicated interval about the y-axis.

   $$y = 9 - x^2\ ,\quad 0 \leq x \ \leq 3$$

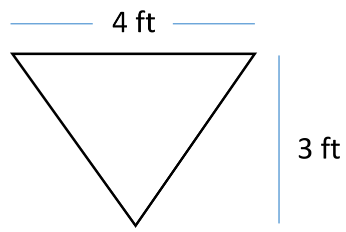
4. Use integration to solve the following application problems involving Work, Centroids, Fluid Pressure, and Fluid Force.

   a. Eighteen foot-pounds of work is required to stretch a spring 4 inches from its natural length. Find the Work required to stretch the spring an additional 3 inches.

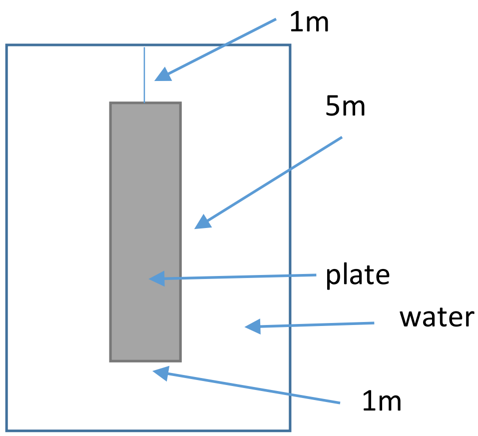
   b. Find $M_x\ , M_y, and\ (\bar{x}, \bar{y})$ for the lamina of uniform density ρ bounded by the graphs of the equations.

   $$y = \sqrt{x} + 1, \qquad y = \frac{1}{3}x + 1$$

c. Find the Fluid Force on the vertical side of the triangular water-filled tank, where the dimensions are given in feet.



d. Find the Fluid Force on the rectangular vertical plate submerged in water, where the dimensions are given in meters and the weight-density of water is 9800 $^{N}/_{m^3}$.



5. Evaluate the following Integrals –

a. $\int t^3\sqrt{t^4+1}\ dt$

b. $\int \frac{3x}{x+4}\ dx$

c. $\int e^{4x}\cos(2x)dx$

d. $\int \frac{lnx}{x^3}dx$

e. $\int \sin^7(2x)\cos(2x)\ dx$

f. $\int \sec^5 x \tan^3 x\ dx$

g. $\int \frac{x^3}{\sqrt{x^2-25}}dx$

h. $\int \frac{\sqrt{25x^2+4}}{x^4}dx$

i. $\int \frac{x^3-x+3}{x^2+x-2}dx$

j. $\int \frac{x^2-6x+2}{x^3+2x^2+x}dx$

6. Determine whether the improper integral converges or diverges. Evaluate the integral if it converges.

a. $\int_0^8 \frac{3}{\sqrt{8-x}}dx$

b. $\int_1^\infty \frac{3}{\sqrt[3]{x}}dx$

7. Test the following series for convergence. Show your work and **state which test** you are using.

a. $\sum_{n=1}^{\infty} \frac{(n+1)!}{5n!}$

b. $\sum_{n=1}^{\infty} e^{-n}$

c. $\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$

d. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

e. $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

f. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-5}$

g. $\sum_{n=1}^{\infty} \left(\frac{n-2}{5n+1}\right)^n$

h. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

8. Find the interval of convergence for each power series. Be sure to check for convergence at the endpoints.

a.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}x^n}{6^n}$$

b.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$$

9. Solve the following applications of geometric series, power series, and/or Taylor/Maclaurin series.

a. Find a geometric power series for the function centered at c = 0 and determine the interval of convergence.

$$f(x) = \frac{5}{4-x^2}$$

b. Use integration to find a power series for the function centered at c = 0 and determine the interval of convergence.

$$f(x) = \ln(x+1) = \int \frac{1}{x+1}\,dx$$

c. Use differentiation to find a power series for the function centered at c = 0 and determine the interval of convergence.

$$f(x) = \frac{-1}{(x+1)^2}dx \quad = \frac{d}{dx}\left[\frac{1}{x+1}\right]$$

10. Find the center, foci, vertice(s), eccentricity, directrix, and asymptotes as appropriate for the given conic sections. Graph the conic section. *Hint: Remember standard form!*

a. $x^2 + 4x + 4y - 4 = 0$

b. $x^2 + 10y^2 - 6x + 20y + 18 = 0$

c. $9x^2 - y^2 - 36x - 6y + 18 = 0$

11. Eliminate the parameter to write the corresponding rectangular equation. Sketch the curve represented by the parametric equations.

$x = 8\cos\theta \quad and \quad y = 8\sin\theta$

12. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Find the slope and concavity (if possible) at the given value of the parameter.

$x = t^2 + 5t + 4, \qquad y = 4t, \qquad t = 0$

13. Find all points (if any) of horizontal and vertical tangency to the curve.

$x = \cos\theta$, $\quad\quad y = 2\sin 2\theta$

14. Perform the following conversions between rectangular and polar form.

a. Find the corresponding rectangular coordinates for the given polar coordinates. $\left(-4, \ \frac{-3\pi}{4}\right)$.

b. Given the rectangular coordinate for a point $\left(-2\sqrt{2}, -2\sqrt{2}\right)$, find **two sets** of polar coordinates of the point for $0 \le \theta < 2\pi$.

c. Convert the rectangular equation to polar form and describe what the graph would look like.

$x^2 - y^2 = 9$

d. Convert the polar equation to a rectangular equation and describe what the graph would look like.

$r = 3\sin\theta$

15. a. Sketch a graph of the polar equation $r = 4(1 - \sin\theta)$ and find the tangent line(s) of the curve at $\theta = \frac{\pi}{2}$.

b. Sketch a graph of the polar equation $r = 3 - 2\cos\theta$.

16. a. Find the area of two petals of the curve $r = 4\sin 3\theta$.
b. Find the area inside of $r = 2\cos\theta$ and outside of $r = 1$.

17. Find the length of the curve over the given interval. $r = 4\sin\theta$, $[0, \pi]$

ANSWER KEY

1a. 4

1b. $\frac{4}{3} - \frac{4}{\pi}$

2a. $\frac{108}{5}\pi$

2b. $\frac{\pi}{6}$

3a. $\frac{20539}{2240} \approx 9.168$

3b. $\frac{\pi}{6}\left(\sqrt{37^3} - 1\right) \approx 117.319$

4a. $37.125\ ft - lbs$

4b. $M_x = \frac{45}{4}\rho, \quad M_y = \frac{81}{5}\rho, \quad \left(\frac{18}{5}, \frac{5}{2}\right)$

4c. $374.4\ lbs$

4d. $171{,}500\ Newtons$

5a. $\frac{1}{6}(t^4+1)^{3/2} + C$

5b. $3x - 12ln|x+4| + C$ *(by Integration Rules)*

5c. $\frac{1}{5}e^{4x}\cos 2x + \frac{1}{10}e^{4x}\sin 2x + C$

5d. $\frac{-1}{2x^2}\ln x - \frac{1}{4x^2} + C$ *(by Parts)*

5e. $\frac{1}{16}\sin^8 2x + C$

5f. $\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$ *(by Trig Integrals)*

5g. $\frac{1}{3}\sqrt{x^2-25}\,(x^2+50) + C$

5h. $-\frac{\sqrt{(25x^2+4)^3}}{12x^3} + C$ *(by Trig Substitution)*

5i. $\frac{x^2}{2} - x + ln|x^2+x-2| + C$

5j. $ln\left|\frac{x^2}{x+1}\right| + \frac{9}{x+1} + C$ *(by Partial Fractions)*

6a. $Converges,\ 12\sqrt{2}$

6b. $Diverges$

7a. $Diverges\ by\ the\ nth\ term\ test$

7b. $Converges\ by\ Geometric\ Series\ \left(|r| = \frac{1}{e} < 1\right)\ \ OR\ by\ Integral\ test$

7c. $Diverges\ by\ the\ Integral\ Test$

7d. $Diverges\ by\ p - Series\ \left(p = \frac{1}{5} < 1\right)$

7e. $Converges - Telescoping\ series$

7f. $Alternating\ Series\ converges\ absolutely\ by\ limit\ comparison\ to\ a\ p - Series.$

7g. $Converges\ by\ the\ Root\ Test\ \left(\frac{1}{5} < 1\right)$

7h. $Converges\ by\ the\ Ratio\ Test\ \ (0 < 1)$

8a. $Converges\ for -6 < x < 6$

8b. $Converges\ for\ 0 < x \le 2$

9a. $\sum_{n=0}^{\infty} 5\,\frac{x^{2n}}{4^{n+1}}\quad (-2, 2)$

9b. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}\quad (-1, 1]$

9c. $\sum_{n=0}^{\infty} (-1)^{n+1}(n+1)x^n\quad (-1, 1)$

#10	Type	Center	Foci	Vertice(s)	Eccent.	Directrix	Asympt
a	Parabola	NA	(-2, 1)	(-2, 2)	NA	y = 3	NA
b	Ellipse	(3, -1)	$\left(3+\frac{3}{\sqrt{10}},-1\right)$ $\left(3-\frac{3}{\sqrt{10}},-1\right)$	(4, -1) (2, -1)	$3/\sqrt{10}$	NA	NA
c	Hyperbola	(2, -3)	$\left(2+\sqrt{10},-3\right)$ $\left(2-\sqrt{10},-3\right)$	(1 , -3) (3 , -3)	$\sqrt{10}$	NA	y = 3x-9 y = -3x+3

11. $x^2+y^2=64,\quad \textit{Circle centered at origin with radius of } 8.$

12. $\frac{dy}{dx}=\frac{4}{2t+5}\ ,\quad \frac{d^2y}{dx^2}=\frac{-8}{(2t+5)^3}\ ,\quad Slope=\frac{4}{5}\ ,\quad Concavity=\frac{-8}{125}\quad (Concave\ Down)$

13. $Horiz.\ Tang.\ at\ \theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\rightarrow\left(\frac{\sqrt{2}}{2},2\right),\left(-\frac{\sqrt{2}}{2},-2\right),\left(-\frac{\sqrt{2}}{2},2\right),\left(\frac{\sqrt{2}}{2},-2\right)$

$Vertical\ Tangents\ at\ \theta=0\ ,\ \pi\rightarrow(1,0),(-1,0)$

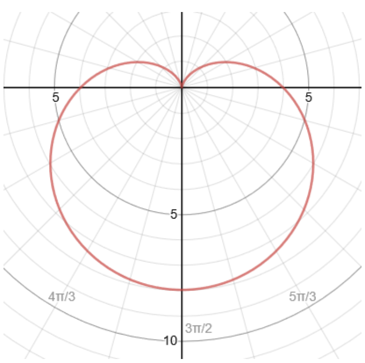
14a. $\left(2\sqrt{2},\ 2\sqrt{2}\right)$

14b. $\left(4,\frac{5\pi}{4}\right)$ and $\left(-4,\frac{\pi}{4}\right)$

14c. $r^2\cos 2\theta=9\ \ (Hyperbola)$

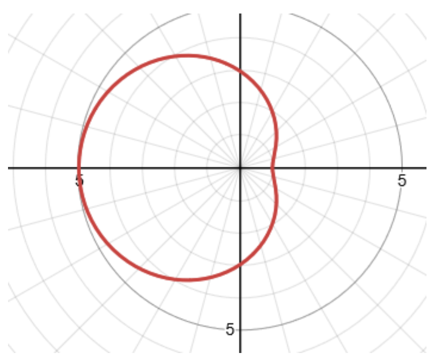
14d. $x^2+\left(y-\frac{3}{2}\right)^2=\frac{9}{4}\quad (Circle)$

15a.



No tangent at pole.

15b.



16. a. $Area:\ A=\frac{8\pi}{3}$

16b. $Area:\ A=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

17. $Length:\ \ s=4\pi$