Logic and Truth Tables

Truth tables are logical devices that predominantly show up in Mathematics, Computer Science, and Philosophy applications. They are used to determine the truth or falsity of propositional statements by listing all possible outcomes of the truth-values for the included propositions.

Proposition - A sentence that makes a claim *(can be an assertion or a denial)* that may be either true or false.

Examples – "Roses are beautiful." →

This is a **Proposition** – It is a *complete sentence* and *makes a claim*. The claim may or may not be true.

"Did you like the movie?" →

This is **NOT** a proposition – It is a question and does *not assert or deny anything*.

Conjunction – an "and" statement. Given two propositions, p and q, "*p and q*" forms a conjunction. The conjunction "*p and q*" is only true if both p and q are true. The truth table can be set up as follows…

This symbol can be used to represent "and".

Truth Table for Conjunction "p and q" (p ∧ q)		
p	***q***	***p and q***
True	True	True
True	False	False
False	True	False
False	False	False

Examples – Determine whether the Conjunction is True or False.

a. The capital of Ireland is Dublin **and** penguins live in Antarctica.

This Conjunction is True because both of the individual propositions are true.

b. A square is a quadrilateral **and** fish are reptiles.

This Conjunction is False because the second proposition is false. Fish are not reptiles.

Disjunction – an "or" statement. Given two propositions, p and q, "*p or q*" forms a disjunction. The disjunction "*p or q*" is true if either p or q is true or if both are true. The disjunction is false only if both p and q are both false. The truth table can be set up as follows…

This symbol can be used to represent "or".

Truth Table for Disjunction "p or q" (p ∨ q)		
p	**q**	**p or q**
True	True	True
True	False	True
False	True	True
False	False	False

Examples - Determine whether the Disjunction is True or False.

a. A triangle has 3 sides **or** 4 sides.

This Disjunction is True because the first proposition is true. Even though the second proposition is false, only one of them needs to be true for the disjunction to be true.

b. All men are tall **or** all women are short.

This Disjunction is False because both propositions are false. Not all men are tall, and not all women are short.

Conditional Propositions – A statement that proposes something is true on the condition that something else is true. For example, "*If p then q*"* , where p is the hypothesis (*antecedent*) and q is the conclusion (*consequent*).

Truth Table for Conditional "if p then q"		
p	***q***	***If p, then q***
True	True	True
True	False	False
False	True	True
False	False	True

*Alternate wording for Conditionals: *"q if p", "p implies q", "q whenever p", "q is necessary for p", "p will lead to q", "p is sufficient for q".*

Examples – Determine whether the Conditional Proposition is True or False.

a. If dolphins swim in the ocean, then birds fly in the sky.

Both the hypothesis and the conclusion are true, so the Conditional Proposition is **True**.

b. If Los Angeles is in Oregon, then the Mississippi river flows backwards.

The hypothesis is false, so the Conditional Proposition is **True** regardless of whether the conclusion is true or not.

c. If Jupiter is a planet, then there are not any volcanoes on the earth.

The hypothesis is true, but the conclusion is false, therefore the Conditional Proposition is **False**.

d. If the Nile River is in South America, then the Amazon River is in Africa.

The hypothesis is false, so the Conditional Proposition is **True** regardless of whether the conclusion is true or not.

Variations on the Conditional - The Converse, Inverse, and Contrapositive are variations on the Conditional proposition.

$\sim \rightarrow$ ***symbol meaning "not"***

Converse **Inverse** **Contrapositive**

Truth Table for Conditional Variations

p	q	*not* p ($\sim p$)	*not* q ($\sim q$)	If p, then q	If q, then p	If *not* p, then *not* q	If *not* q, then *not* p
True	True	False	False	True	True	True	True
True	False	False	True	False	True	True	False
False	True	True	False	True	False	False	True
False	False	True	True	True	True	True	True

Logically Equivalent - Statements are logically equivalent if they share the same truth tables. Therefore, a Conditional statement and its Contrapositive are logically equivalent. The Inverse and Converse are also logically equivalent to each other.

Examples –

a. For the given Conditional Statement, write the Converse, Inverse, and Contrapositive statements.

CONDITIONAL

If my favorite football team qualifies for the SuperBowl,
then I will buy tickets to the game.

CONVERSE

If I buy tickets to the game,
then my favorite football team will qualify for the SuperBowl.

INVERSE

If my favorite football team does **not** qualify for the SuperBowl,
then I will **not** buy tickets to the game.

CONTRAPOSITIVE

If I do **not** buy tickets to the game,
then my favorite football team will **not** qualify for the SuperBowl.

Try these on your own!

I. Determine whether the Statement is a Proposition (*Yes*) or not (*No*).

a. Mathematics is easy. *(Answer: Yes)*
b. What is the temperature outside? *(Answer: No)*
c. Rock climbing is fun! *(Answer: Yes)*

II. Determine whether each Conjunction/Disjunction is True or False.

a. Horses are mammals and frogs are amphibians. *(Answer: True)*
b. Mark Twain was a famous athlete or actor. *(Answer: False)*
c. All birds cannot fly or all mammals can swim. *(Answer: True)*

III. Determine whether each Conditional Proposition is True or False.

a. If triangles are polygons, then circles are ellipses. *(Answer: True)*
b. If 2x4=8, then 2+4=8. *(Answer: False)*
c. If Babe Ruth was in the NFL, then Madonna played in the NHL. (Answer: True)

IV. Write the Converse, Inverse, and Contrapositive for the Conditional Proposition: *If I save enough money, then I will go on vacation.*