Math 190 (Calculus II) Final Review

1. Sketch the region enclosed by the given curves and find the area of the region.
   
   a. \( y = 7 - 2x^2 \), \( y = x^2 + 4 \)
   
   b. \( y = \cos \left( \frac{\pi x}{2} \right) \), \( y = 1 - x^2 \)

2. Use the specified method to find the volume generated by rotating the region bounded by the given curves and axis of rotation.
   
   a. Use the Disk/Washer Method: \( y = 4 - x^2 \), \( y = 2 - x \), About \( x \)-axis
   
   b. Use the Shell Method: \( y = 2x - x^2 \), \( y = x \), About the line \( x = 1 \)

3. a. Find the Arc Length of the graph of the function over the indicated interval.

   \[ y = \frac{x^7}{14} + \frac{1}{10x^5} \], \([1, 2]\)

   b. Find the Surface Area generated by revolving the curve on the indicated interval about the \( y \)-axis.

   \( y = 9 - x^2 \), \( 0 \leq x \leq 3 \)

4. Use integration to solve the following application problems involving Work, Centroids, Fluid Pressure, and Fluid Force.

   a. Eighteen foot-pounds of work is required to stretch a spring 4 inches from its natural length. Find the Work required to stretch the spring an additional 3 inches.

   b. Find \( M_x \), \( M_y \), and \((\bar{x}, \bar{y})\) for the lamina of uniform density \( \rho \) bounded by the graphs of the equations.

   \[ y = \sqrt{x} + 1 \], \( y = \frac{1}{3}x + 1 \)
c. Find the Fluid Force on the vertical side of the triangular water-filled tank, where
the dimensions are given in feet.

![Triangular tank diagram]

4 ft

3 ft

d. Find the Fluid Force on the rectangular vertical plate submerged in water, where
the dimensions are given in meters and the weight-density of water is 9800
\( N/m^3 \).

![Rectangular plate diagram]

5 m

1 m

plate

water

5. Evaluate the following Integrals –

a. \( \int t^3\sqrt{t^4 + 1} \, dt \)

b. \( \int \frac{3x}{x+4} \, dx \)

c. \( \int e^{4x} \cos(2x) \, dx \)

d. \( \int \frac{\ln x}{x^3} \, dx \)

e. \( \int \sin^7(2x)\cos(2x) \, dx \)

f. \( \int \sec^5 x \tan^3 x \, dx \)

g. \( \int \frac{x^3}{\sqrt{x^2-25}} \, dx \)

h. \( \int \frac{\sqrt{25x^2+4}}{x^4} \, dx \)

i. \( \int \frac{x^3-x+3}{x^2+x-2} \, dx \)

j. \( \int \frac{x^2-6x+2}{x^3+2x^2+x} \, dx \)
6. Determine whether the improper integral converges or diverges. Evaluate the integral if it converges.

a. \[ \int_{0}^{8} \frac{3}{\sqrt{8-x}} \, dx \]

b. \[ \int_{1}^{\infty} \frac{3}{\sqrt{x}} \, dx \]

7. Test the following series for convergence. Show your work and state which test you are using.

a. \[ \sum_{n=1}^{\infty} \frac{(n+1)!}{5n!} \]

b. \[ \sum_{n=1}^{\infty} e^{-n} \]

c. \[ \sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1} \]

d. \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]

e. \[ \sum_{n=1}^{\infty} \frac{3}{n(n+3)} \]

f. \[ \sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3 - 5} \]

g. \[ \sum_{n=1}^{\infty} \left( \frac{n-2}{5n+1} \right)^n \]

h. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} \]

8. Find the interval of convergence for each power series. Be sure to check for convergence at the endpoints.

a. \[ \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{6^n} \]

b. \[ \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x - 1)^{n+1}}{n + 1} \]
9. Solve the following applications of geometric series, power series, and/or Taylor/Maclaurin series.

a. Find a geometric power series for the function centered at c = 0 and determine the interval of convergence.

\[ f(x) = \frac{5}{4-x^2} \]

b. Use integration to find a power series for the function centered at c = 0 and determine the interval of convergence.

\[ f(x) = \ln(x+1) = \int \frac{1}{x+1} \, dx \]

c. Use differentiation to find a power series for the function centered at c = 0 and determine the interval of convergence.

\[ f(x) = \frac{-1}{(x+1)^2} \, dx = \frac{d}{dx} \left[ \frac{1}{x+1} \right] \]

10. Find the center, foci, vertex(s), eccentricity, directrix, and asymptotes as appropriate for the given conic sections. Graph the conic section. *Hint: Remember standard form!*

a. \( x^2 + 4x + 4y - 4 = 0 \)

b. \( x^2 + 10y^2 - 6x + 20y + 18 = 0 \)

c. \( 9x^2 - y^2 - 36x - 6y + 18 = 0 \)

11. Eliminate the parameter to write the corresponding rectangular equation. Sketch the curve represented by the parametric equations.

\[ x = 8 \cos \theta \quad \text{and} \quad y = 8 \sin \theta \]

12. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \). Find the slope and concavity (if possible) at the given value of the parameter.

\[ x = t^2 + 5t + 4, \quad y = 4t, \quad t = 0 \]
13. Find all points (if any) of horizontal and vertical tangency to the curve.

\[ x = \cos \theta, \quad y = 2 \sin 2\theta \]

14. Perform the following conversions between rectangular and polar form.

a. Find the corresponding rectangular coordinates for the given polar coordinates. 
   \( (-4, \frac{-3\pi}{4}) \).

b. Given the rectangular coordinate for a point \((-2\sqrt{2}, -2\sqrt{2})\), find two sets of polar coordinates of the point for \(0 \leq \theta < 2\pi\).

c. Convert the rectangular equation to polar form and describe what the graph would look like.
   \[ x^2 - y^2 = 9 \]

d. Convert the polar equation to a rectangular equation and describe what the graph would look like.
   \[ r = 3 \sin \theta \]

15. a. Sketch a graph of the polar equation \( r = 4(1 - \sin \theta) \) and find the tangent line(s) of the curve at \( \theta = \frac{\pi}{2} \).

b. Sketch a graph of the polar equation \( r = 3 - 2 \cos \theta \).

16. a. Find the area of two petals of the curve \( r = 4 \sin 3\theta \).

b. Find the area inside of \( r = 2 \cos \theta \) and outside of \( r = 1 \).

17. Find the length of the curve over the given interval. \( r = 4 \sin \theta, \quad [0, \pi] \)
**ANSWER KEY**

1a. \(4\) 

1b. \(\frac{4}{3} - \frac{4}{\pi}\)

2a. \(\frac{108}{5}\) \(\pi\) 

2b. \(\frac{\pi}{6}\)

3a. \(\frac{20539}{2240} \approx 9.168\) 

3b. \(\frac{\pi}{6} \left(\sqrt{37} - 1\right) \approx 117.319\)

4a. 37.125 \(ft - lbs\) 

4b. \(M_x = \frac{45}{4} \rho, \quad M_y = \frac{81}{5} \rho, \quad \left(\frac{18}{5}, \frac{5}{2}\right)\)

4c. 374.4 \(lbs\) 

4d. 171,500 \(Newtons\)

5a. \(\frac{1}{6} \left(t^4 + 1\right)^{3/2} + C\) 

5b. \(3x - 12\ln|x + 4| + C\) (by Integration Rules)

5c. \(\frac{1}{5} e^{4x} \cos 2x + \frac{1}{10} e^{4x} \sin 2x + C\) 

5d. \(-\frac{1}{2x^2} \ln x - \frac{1}{4x^2} + C\) (by Parts)

5e. \(\frac{1}{16} \sin^8 2x + C\)

5f. \(\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C\) (by Trig Integrals)

5g. \(\frac{1}{3} \sqrt{x^2 - 25} (x^2 + 50) + C\)

5h. \(-\frac{\sqrt{25x^2 + 4}}{12x^3} + C\) (by Trig Substitution)

5i. \(\frac{x^2}{2} - x + \ln|x^2 + x - 2| + C\)

5j. \(\ln \left|\frac{x^2}{x+1}\right| + \frac{9}{x+1} + C\) (by Partial Fractions)

6a. **Converges**, \(\sqrt{2}\) 

6b. **Diverges**

7a. **Diverges by the nth term test**

7b. **Converges by Geometric Series** \(|r| = \frac{1}{e} < 1\) \(\text{OR by Integral test}\)

7c. **Diverges by the Integral Test**

7d. **Diverges by p-Series** \((p = \frac{1}{5} < 1)\)

7e. **Converges – Telescoping series**

7f. **Alternating Series converges absolutely by limit comparison to a p – Series.**

7g. **Converges by the Root Test** \(\left(\frac{1}{5}\right) < 1\)

7h. **Converges by the Ratio Test** \((0 < 1)\)

8a. **Converges for** \(-6 < x < 6\) 

8b. **Converges for** \(0 < x \leq 2\)

9a. \(\sum_{n=0}^{\infty} 5\frac{x^{2n}}{4^{n+1}} \left(-2, 2\right)\)

9b. \(\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} \left(-1, 1\right)\)

9c. \(\sum_{n=0}^{\infty} (-1)^{n+1} (n+1)x^n \left(-1, 1\right)\)
<table>
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<tr>
<th>#10</th>
<th>Type</th>
<th>Center</th>
<th>Foci</th>
<th>Vertice(s)</th>
<th>Eccent.</th>
<th>Directrix</th>
<th>Asympt</th>
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<tbody>
<tr>
<td>a</td>
<td>Parabola</td>
<td>NA</td>
<td>(-2, 1)</td>
<td>(-2, 2)</td>
<td>NA</td>
<td>y = 3</td>
<td>NA</td>
</tr>
<tr>
<td>b</td>
<td>Ellipse</td>
<td>(3, -1)</td>
<td>(\left(\frac{3}{\sqrt{10}}, -1\right))(\left(\frac{-3}{\sqrt{10}}, -1\right))</td>
<td>(4, -1)</td>
<td>(\frac{3}{\sqrt{10}})</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>c</td>
<td>Hyperbola</td>
<td>(2, -3)</td>
<td>(\left(2 + \frac{\sqrt{10}}{2}, -3\right))(\left(2 - \frac{\sqrt{10}}{2}, -3\right))</td>
<td>(1, -3)</td>
<td>(\sqrt{10})</td>
<td>NA</td>
<td>y = 3x - 9</td>
</tr>
</tbody>
</table>

11. \(x^2 + y^2 = 64\), \textit{Circle centered at origin with radius of 8.}

12. \(\frac{dy}{dx} = \frac{4}{2t+5}, \ \frac{d^2y}{dx^2} = \frac{-8}{(2t+5)^3}\), Slope = \(\frac{4}{5}\), Concavity = \(-\frac{8}{125}\) (Concave Down)

13. \(\text{Horiz. Tang. at } \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \Rightarrow (\frac{\sqrt{2}}{2}, 2), (-\frac{\sqrt{2}}{2}, -2), (-\frac{\sqrt{2}}{2}, 2), (\frac{\sqrt{2}}{2}, -2)\)

   \(\text{Vertical Tangents at } \theta = 0, \pi \Rightarrow (1,0), (-1,0)\)

14a. \((2\sqrt{2}, 2\sqrt{2})\) 14b. \((4, \frac{5\pi}{4})\) and \((-4, \frac{\pi}{4})\)

14c. \(r^2 \cos 2\theta = 9\) \textit{(Hyperbola)} 14d. \(x^2 + (y - \frac{3}{2})^2 = \frac{9}{4}\) \textit{(Circle)}

15a.  
15b.  

\textit{No tangent at pole.}

16. a. \(\text{Area: } A = \frac{8\pi}{3}\) 16b. \(\text{Area: } A = \frac{\pi}{3} + \frac{\sqrt{3}}{2}\)

17. \(\text{Length: } s = 4\pi\)