Graphing a Quadratic Function: \( f(x) = ax^2 + bx + c \)

**Quadratic Functions are second degree polynomials** (i.e. highest power of the domain variable is 2). Quadratics can be written in several forms - General Form, Standard Form *(also called Vertex Form)*, and Factored form *. The graph of a Quadratic Function is called a Parabola. It’s general shape is curved and looks like a “U”. The “U” is right side up if “a” is positive \((a > 0)\), and it is upside down if “a” is negative \((a < 0)\). The Vertex \((h, k)\) is either the lowest (right side up) or the highest (upside down) point on the parabola. The **Axis of Symmetry** is a vertical line that visually cuts the parabola in half and is written as \(x = h\).

**General Form \((a, b, c \in \mathbb{R})\)**

\[
f(x) = ax^2 + bx + c
\]

The **y-intercept** \((0, c)\) of the graph is easily identifiable from General Form.

The **x-intercept(s) (if any)** can be found by factoring and/or using the quadratic formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

The **Vertex** \((h, k)\), **Min/Max value** \((k)\), and **Axis of Symmetry** \((x = h)\) can be found by completing the square or by using the vertex formula:

\[
h = \frac{-b}{2a}, \quad k = f(h)
\]

**Standard (Vertex) Form \((a, h, k \in \mathbb{R})\)**

\[
f(x) = a(x - h)^2 + k
\]

- The **Vertex** \((h, k)\),
- The **Min/Max value** \((k)\) of the function, and
- The **Axis of Symmetry** \((x = h)\)

are all easily identifiable from Vertex Form.

The **x-intercept(s) (if any)** can be found by using the square root property.

The **y-intercept** can be found by evaluating \(f(0)\).

\[
f(0) = a(0 - h)^2 + k = ah^2 + k
\]

**Parabolic Graph of a Quadratic Function**

- **y-intercept** \((0, c)\)
- **Origin** \((0, 0)\)
- **Distance k** (Up/Down)
- **Distance h** (Rt/Lft) *(from Origin)*
- **Vertex**: \((h, k)\)
  - “k” is the Min or Max value of the function.
  - “h” is the domain value that results in the Min/Max.
- **Axis of Symmetry** \(x = h\)
- **x-intercepts**, *also called real “zeros”*
- **This Parabola is “Face Up”** *(\(a > 0)\)*
**Practice Graphing Quadratic Functions** ➞ \( f(x) = ax^2 + bx + c = a(x - h)^2 + k \)

**Examples:**

**Graph** the following Quadratic given in General Form: \( f(x) = -3x^2 - 6x + 24 \)

**Identify the Vertex:** \((Calculate)\)
\[
h = \frac{-b}{2a} = \frac{-(-6)}{2(-3)} = \frac{6}{-6} = -1
\]
\[
k = f(h) = -3(-1)^2 - 6(-1) + 24 = 27
\]

**Vertex** = \((h, k) = (-1, 27)\)

**Find the x-intercept(s):** \((Factor or use the Quadratic Formula)\)
\[
f(x) = -3x^2 - 6x + 24 = 0
\]
\[
f(x) = -3(x^2 + 2x - 8) = 0
\]
\[
f(x) = -3(x + 4)(x - 2) = 0
\]
\[
x + 4 = 0 \quad x - 2 = 0
\]
\[
x = -4 \quad x = 2
\]

**x-intercepts:** \((-4, 0), (2, 0)\)

**Find the y-intercept:** \((0, c) = (0, 24)\)

**Find the axis of symm:** \(x = -1\)

**Extra Points:** Use point plotting if needed.

**Graph** the following Quadratic given in Standard (Vertex) Form: \( f(x) = 3(x + 1)^2 - 4 \)

**Identify the Vertex:** \((from the formula)\)
\[
(h, k) = (-1, -4)
\]

**Find the x-intercept(s):** \((Square root property)\)
\[
f(x) = 3(x + 1)^2 - 4 = 0
\]
\[
3(x + 1)^2 = 4
\]
\[
(x + 1)^2 = \frac{4}{3}
\]
\[
\sqrt{(x + 1)^2} = \pm \sqrt{\frac{4}{3}}
\]
\[
x + 1 = \pm \frac{2\sqrt{3}}{3}
\]
\[
x = -1 \pm \frac{2\sqrt{3}}{3}
\]

**x-intercepts:** \((-1 + \frac{2\sqrt{3}}{3}, 0), (-1 - \frac{2\sqrt{3}}{3}, 0)\)

**Find the y-intercept:** \(f(0) = 3(1)^2 - 4 = -1\)
\[
= (0, -1)
\]

**Find the axis of symm:** \(x = -1\)

**Extra Points:** Use point plotting if needed.

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\(a < 0,\) So facing DOWN
- Vertex (Max)
- y-intercept
- x-intercepts
- Axis of Symmetry
- Range: \((-\infty, k]\)
- Domain: \((-\infty, \infty)\)

\(a > 0,\) So facing UP
- x-intercepts
- y-intercept
- Vertex (Min)
- Axis of Symmetry
- Range: \([k, \infty)\)
- Domain: \((-\infty, \infty)\)
Practice Problems: Try these on your own!

Graph the following Quadratic Functions given in General Form. Find the Vertex, y-intercept, and x-intercept(s) if they exist. State the Domain and the Range. Also find and show the Axis of Symmetry. State whether the parabola opens up or down.

1. \( f(x) = -x^2 + x + 6 \)
   \[ \text{Answer:} \]
   \[ \text{Vertex: } \left( \frac{1}{2}, \frac{25}{4} \right) \]
   \[ \text{y-intercept: } (0, 6) \]
   \[ \text{x-intercept(s): } (-2, 0), \ (3, 0) \]
   \[ \text{Axis of Symmetry: } x = \frac{1}{2} \]
   \[ \text{Domain: } \mathbb{R} \text{ or } (-\infty, \infty) \]
   \[ \text{Range: } \left( -\infty, \frac{25}{4} \right] \]
   \[ \text{Opens: Down} \]

2. \( f(x) = 2x^2 + 4x + 4 \)
   \[ \text{Answer:} \]
   \[ \text{Vertex: } (-1, 2) \]
   \[ \text{y-intercept: } (0, 4) \]
   \[ \text{x-intercept(s): None} \]
   \[ \text{Axis of Symmetry: } x = -1 \]
   \[ \text{Domain: } \mathbb{R} \text{ or } (-\infty, \infty) \]
   \[ \text{Range: } [2, \infty) \]
   \[ \text{Opens: Up} \]
Graph the following Quadratic Functions given in Standard (Vertex) Form. Find the Vertex, y-intercept, and x-intercept(s) if they exist. State the Domain and the Range. Also find and show the Axis of Symmetry. State whether the parabola opens up or down.

3. \( f(x) = 2(x - 2)^2 - 3 \)
   \[ \text{Answer:} \]
   \[
   \begin{align*}
   \text{Vertex:} & \quad (2, -3) \\
   \text{y-intercept:} & \quad (0, 5) \\
   \text{x-intercept(s):} & \quad \left(\frac{4 + \sqrt{5}}{2}, 0\right), \left(\frac{4 - \sqrt{5}}{2}, 0\right) \\
   \text{Axis of Symmetry:} & \quad x = 2 \\
   \text{Domain:} & \quad \mathbb{R} \text{ or } (-\infty, \infty) \\
   \text{Range:} & \quad [-3, \infty) \\
   \text{Opens:} & \quad \text{Up}
   \end{align*}
   \]

4. \( f(x) = -\frac{1}{4}(x + 4)^2 - 2 \)
   \[ \text{Answer:} \]
   \[
   \begin{align*}
   \text{Vertex:} & \quad (-4, -2) \\
   \text{y-intercept:} & \quad (0, -6) \\
   \text{x-intercept(s):} & \quad \text{None} \\
   \text{Axis of Symmetry:} & \quad x = -4 \\
   \text{Domain:} & \quad \mathbb{R} \text{ or } (-\infty, \infty) \\
   \text{Range:} & \quad (-\infty, -2] \\
   \text{Opens:} & \quad \text{Down}
   \end{align*}
   \]