Solving Literal Equations

Literal Equations – Equations with multiple variables where you are asked to solve for just one of the variables. (Usually represent formulas used in the sciences and/or geometry)

To solve literal equations: Use the same process you use to isolate the variable in an algebraic equation with one variable. It’s just that you are going to be adding, subtracting, multiplying, and dividing (and sometimes factoring) variables as well as numbers.

CAUTION: BE CAREFUL NOT TO COMBINE UNLIKE TERMS!

Example 1: Solve $E = IR$ for $R$.

Goal: Isolate $R$ to get $R = \text{an expression in } E \text{ and } I$

To isolate $R$, divide both sides of the equation by $I$:

$$\frac{E}{I} = \frac{IR}{I}$$

Simplify:

$$\frac{E}{I} = R$$

Solution: $R = \frac{E}{I}$

Example 2: Solve $\frac{d}{t} = r$ for $t$.

Goal: Isolate $t$ to get $t = \text{an expression in } d \text{ and } r$

First multiply both sides of the equation by $t$ to clear the fractions:

$$\frac{d}{t} \cdot t = r(t)$$

Simplify:

$$d = rt$$

To isolate $t$, divide both sides of the equation by $r$:

$$\frac{d}{r} = \frac{rt}{r}$$

Simplify:

$$\frac{d}{r} = t$$

Solution: $t = \frac{d}{r}$

Example 3: Solve $A = \frac{1}{2} h(b_1 + b_2)$ for $b_1$

Goal: Isolate $b_1$ to get $b_1 = \text{an expression in } A, h, \text{ and } b_2$ (Note: $b_1$ and $b_2$ are two different variables.)

First multiply both sides of the equation by 2 to clear the fractions:

$$2A = \left(\frac{2}{1}\right) \frac{1}{2} h(b_1 + b_2)$$

(continued on next page)
Simplify:
\[ 2A = h(b_1 + b_2) \]
Distribute \( h \):
\[ 2A = hb_1 + hb_2 \]

Next subtract \( hb_2 \) from both sides of the equation to get \( hb_1 \) alone:
\[
\begin{align*}
2A &= \quad hb_1 + \frac{hb_2}{h} \\
-hb_2 &= -\frac{hb_2}{h} \\
2A - hb_2 &= hb_1
\end{align*}
\]

To isolate \( b_1 \), divide both sides of the equation by \( h \):
\[
\begin{align*}
\frac{2A - hb_2}{h} &= \frac{hb_1}{h} \\
\frac{2A - hb_2}{h} &= b_1
\end{align*}
\]

Solution: \( b_1 = \frac{2A - hb_2}{h} \)

Example 4:
Solve \( I = \frac{PN}{RN+A} \) for \( N \)

Goal: Isolate \( N \) to get \( N = \) an expression in \( I, P, R, \) & \( A \):

First multiply both sides of the equation by \((RN+A)\) to clear the fractions:
\[
(RN + A)I = \frac{PN}{RN+A}(RN + A)
\]

Simplify:
\[
(RN + A)I = PN
\]

Distribute \( I \):
\[
IRN + IA = PN
\]

Subtract \( IRN \) from both sides to get all \( N \)'s on the same side:
\[
\begin{align*}
IRN + IA &= \quad PN \\
-IRN &= -IRN
\end{align*}
\]

Note: \( PN \) & \( IRN \) are not like terms we cannot combine them!
\[
IA = PN - IRN
\]

But we can factor out the \( N \) from each term!
\[
IA = N(P - IR)
\]

Finally, we can divide both sides by \((P - IR)\) to isolate \( N \):
\[
\begin{align*}
\frac{IA}{P-IR} &= \frac{N(P-IR)}{P-IR} \\
\frac{IA}{P-IR} &= \frac{N}{P-IR}
\end{align*}
\]

Simplify:
\[
\frac{IA}{P-IR} = N
\]

Solution: \( N = \frac{IA}{P-IR} \)
Practice Problems

1. Solve $d = rt$ for $r$
2. Solve $P = \frac{144p}{y}$ for $p$
3. Solve $R = \frac{Cs}{d}$ for $C$
4. Solve $P = a + b + c$ for $b$
5. Solve $T = m - n$ for $n$
6. Solve $A = \frac{a + b}{2}$ for $b$
7. Solve $V = lwh$ for $w$
8. Solve $m = \frac{y_2 - y_1}{x_2 - x_1}$ for $y_2$
9. Solve $ax + by = c$ for $y$
10. Solve $A = \frac{a + b + c + d}{4}$ for $c$
11. Solve $S = 2(lw + lh + wh)$ for $w$
12. Solve $P = 2(l + w)$ for $l$
13. Solve $d = \frac{c}{\pi}$ for $\pi$
14. Solve $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$ for $f$
15. Solve $A = p(1 + rt)$ for $t$
16. Solve $l = prt$ for $r$
17. Solve $ax + b = c$ for $a$
18. Solve $S = 2\pi rh$ for $h$
19. Solve $A = 2\pi r^2 + 2\pi rh$ for $h$
20. Solve $y - y_1 = m(x - x_1)$ for $x$
21. Solve $R = \frac{l + 3w}{2}$ for $w$
22. Solve $ax + by + c = 0$ for $y$
23. Solve $C = \frac{5}{9}(F - 32)$ for $F$
24. Solve $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for $R$
25. Solve $H = \frac{62.4 N S}{33,000}$ for $N$
26. Solve $B = \frac{703w}{h^2}$ for $w$
27. Solve $K = \frac{1}{2}mv^2$ for $m$
28. Solve $5t - 2r = 25$ for $t$
29. Solve $S = R - rR$ for $R$
30. Solve $V = \frac{1}{3}\pi h^2(3r - h)$ for $r$
31. Solve $A = \frac{1}{2}nal$ for $n$
32. Solve $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ for $T_1$
33. Solve $F = \frac{gmi_m}{d^2}$ for $g$
34. Solve $\frac{12ds}{w} = CD$ for $w$
35. Solve $A = \frac{1}{2}bh$ for $b$
36. Solve $s = r\theta$ for $\theta$
37. Solve $h = vt - 16t^2$ for $v$
38. Solve $C = \frac{100B}{L}$ for $L$
39. Solve $A = S(1 - DN)$ for $N$
40. Solve $D = \frac{11}{5}(P - 15)$ for $P$
41. Solve $E = IR$ for $I$
42. Solve $E = mc^2$ for $c^2$
43. Solve $F = \frac{lt}{d}$ for $l$
44. Solve $A = 2\pi r^2 + 2\pi rh$ for $\pi$
1. \( r = \frac{d}{t} \)
2. \( p = \frac{Py}{144} \)
3. \( C = \frac{Rd}{S} \)
4. \( b = P - a - c \)
5. \( n = m - T \)
6. \( b = 2A - a \)
7. \( w = \frac{v}{l h} \)
8. \( y_2 = mx_2 - mx_1 + y_1 \)
9. \( y = \frac{c - ax}{b} \)
10. \( c = 4A - a - b - d \)
11. \( w = \frac{S - 2lh}{2l + 2h} \)
12. \( l = \frac{P - 2w}{2} \)
13. \( \pi = \frac{c}{d} \)
14. \( f = \frac{ab}{b + a} \)
15. \( t = \frac{A - p}{pr} \)
16. \( r = \frac{l}{pt} \)
17. \( a = \frac{c - b}{x} \)
18. \( h = \frac{S}{2\pi r} \)
19. \( h = \frac{A - 2\pi r^2}{2\pi r} \)
20. \( x = \frac{y - y_1 + mx_1}{m} \)
21. \( w = \frac{2R - l}{3} \)
22. \( y = \frac{-ax - c}{b} \)
23. \( F = \frac{9}{5} C + 32 \)
24. \( R = \frac{R_1 R_2}{R_2 + R_1} \)
25. \( N = \frac{33,000H}{62.4S} \)
26. \( w = \frac{Bh^2}{703} \)
27. \( m = \frac{2k}{v^2} \)
28. \( t = \frac{2}{5} r + 5 \)
29. \( R = \frac{S}{1 - r} \)
30. \( r = \frac{3V + \pi h^3}{3\pi h^2} \)
31. \( n = \frac{2A}{al} \)
32. \( T_1 = \frac{T_2 P_1 V_1}{P_2 V_2} \)
33. \( g = \frac{Fd^2}{m_1 m_2} \)
34. \( w = \frac{12ds}{CD} \)
35. \( b = \frac{2A}{h} \)
36. \( \theta = \frac{s}{r} \)
37. \( v = \frac{h + 16t^2}{t} \)
38. \( L = \frac{100B}{C} \)
39. \( N = \frac{S - A}{SD} \)
40. \( P = \frac{5}{11} D + 15 \)
41. \( l = \frac{E}{R} \)
42. \( c^2 = \frac{E}{m} \)
43. \( l = \frac{Fd}{t} \)
44. \( \pi = \frac{A}{2r^2 + 2rh} \)