Factoring – “Bottoms Up” Method

If a Trinomial of the form \( ax^2 + bx + c = 0 \) is factorable, it can be completed using the Bottoms Up Method according to the following steps...

Step 1. Make sure the trinomial is in standard form \( (ax^2 + bx + c = 0) \).

Step 2. Factor out a GCF (greatest common factor) if applicable.

Step 3. Multiply \( a \cdot c \) and re-write the polynomial as: \( 1x^2 + bx + ac = 0 \).

Step 4. Factor as normal, by finding the two factors \( (n_1, n_2) \) of \( a \cdot c \) that add up to \( b \).

Step 5. Write the binomial factors as \( (x + n_1)(x + n_2) = 0 \).

Step 6. Divide the constants \( (n_1 and n_2) \) in each binomial factor by the original value of \( a \).

Step 7. Simplify the resulting 2 fractions if applicable.

Step 8. If the simplified fraction has a denominator other than 1, move the denominator to become the coefficient in front of the variable (“bottoms up”).

Step 9. Check the answer - Multiply the answers to verify that you get the original trinomial.

Example 1

Step 1: \( 6x^2 + 5x - 4 = 0 \)

Step 2: No GCF

Step 3: \( a \cdot c = (6)(-4) = -24 \)

Re-write \( x^2 + 5x - 24 = 0 \)

Step 4: Find factors of \(-24\) that add to \( b \) (5)

Factors \( \rightarrow (+8)(-3) \)

Step 5: \( (x + 8)(x - 3) = 0 \)

Step 6: \( \left( x + \frac{8}{6} \right) \left( x - \frac{3}{6} \right) = 0 \)

Divide the constants by the original value of \( a \)

Step 7: \( \left( x + \frac{4}{3} \right) \left( x - \frac{1}{2} \right) = 0 \)

Reduce the resulting fractions

Step 8: \( (3x + 4)(2x - 1) = 0 \)

Move the denominator so that it becomes the coefficient in front of the variable – “bottoms up”

Example 2

Step 1: \( 6x^2 - 21x - 45 = 0 \)

Step 2: \( 3(2x^2 - 7x - 15) = 0 \)

Step 3: \( a \cdot c = (2)(-15) = -30 \)

Re-write \( x^2 - 21x - 30 = 0 \)

Step 4: Find factors of \(-30\) that add to \( b \) (−7)

Factors \( \rightarrow (-10)(3) \)

Step 5: \( 3(x - 10)(x + 3) = 0 \)

Step 6: \( 3 \left( x - \frac{10}{2} \right) \left( x + \frac{3}{2} \right) = 0 \)

Step 7: \( 3(x - 5) \left( x + \frac{3}{2} \right) = 0 \)

Step 8: \( 3(x - 5)(2x + 3) = 0 \)
Directions - Factor the following trinomials by using the “bottoms up” factoring method.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
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<tbody>
<tr>
<td>1. $2x^2 - 9x - 18 = 0$</td>
<td>$(x - 6)(2x + 3) = 0$</td>
</tr>
<tr>
<td>2. $8x^2 + 2x - 3 = 0$</td>
<td>$(2x - 1)(4x + 3) = 0$</td>
</tr>
<tr>
<td>3. $3x^2 + 19x = 40$</td>
<td>$(x + 8)(3x - 5) = 0$</td>
</tr>
<tr>
<td>4. $8x^2 - 12x - 8 = 0$</td>
<td>$4(2x + 1)(x - 2) = 0$</td>
</tr>
<tr>
<td>5. $10x^2 - 25x = 125$</td>
<td>$5(2x + 5)(x - 5) = 0$</td>
</tr>
<tr>
<td>6. $\frac{5}{2}x^2 - \frac{11}{2}x + 1 = 0$</td>
<td>$\frac{1}{2}(5x - 1)(x - 2) = 0$</td>
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