

## MULTIPLYING RATIONAL EXPRESSIONS

To Multiply Two Fractions	
$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad b \neq 0 \quad \text{and} \quad d \neq 0$	

**Example 1** Multiply:  $\left(\frac{3}{5}\right)\left(\frac{-2}{9}\right)$

Solution: First divide out any common factors to both a numerator and a denominator; then multiply.

$$\frac{\cancel{3}^1}{5} \cdot \frac{-2}{\cancel{9}_3} = \frac{1 \cdot (-2)}{5 \cdot 3} = -\frac{2}{15}$$

The same principles apply when multiplying rational expressions containing variables. Before multiplying, you should first divide out any common factors to both a numerator and a denominator.

To Multiply Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors.
3. Multiply numerators together and multiply denominators together.

**Example 2** Multiply:  $\frac{3x^2}{2y} \cdot \frac{4y^3}{3x}$

Solution: 
$$\frac{3x^2}{2y} \cdot \frac{4y^3}{3x} = \frac{\cancel{3}^1 x^2}{\cancel{2}_1 y} \cdot \frac{\cancel{4}^2 y^3}{\cancel{3}_1 x} = 2xy^2$$

**Example 3** Multiply:  $(x-5) \cdot \frac{7}{x^3-5x^2}$

Solution: 
$$(x-5) \cdot \frac{7}{x^3-5x^2} = \frac{\cancel{x-5}}{1} \cdot \frac{7}{x^2(\cancel{x-5})} = \frac{7}{x^2}$$

**Example 4** Multiply:  $\frac{(x+2)^2}{6x^2} \cdot \frac{3x}{x^2-4}$

Solution: 
$$\frac{(x+2)^2}{6x^2} \cdot \frac{3x}{x^2-4} = \frac{(\cancel{x+2})(x+2)}{\cancel{6}^2 x^2} \cdot \frac{\cancel{3}^1 x}{(\cancel{x+2})(x-2)} = \frac{x+2}{2x(x-2)}$$

Note: When multiplying rational expressions, if only the signs differ in a numerator and a denominator (for instance, the numerator is  $x-7$  & the denominator is  $7-x$ ), factor out  $-1$  from *either* the numerator or denominator; then divide out the common factor.

$$\frac{a-b}{x} \cdot \frac{y}{b-a} = \frac{\cancel{a-b}}{x} \cdot \frac{y}{-1(\cancel{a-b})} = -\frac{y}{x}$$

**Example 5** Multiply:  $\frac{3x+2}{2x-1} \cdot \frac{4-8x}{3x+2}$

Solution: 
$$\frac{3x+2}{2x-1} \cdot \frac{4-8x}{3x+2} = \frac{3x+2}{2x-1} \cdot \frac{4(1-2x)}{3x+2} = \frac{\cancel{3x+2}}{2x-1} \cdot \frac{4(-1)(\cancel{2x-1})}{\cancel{3x+2}} = \frac{-4}{1} = -4$$

**Example 6** Multiply:  $\frac{2x^2+7x-15}{4x^2-8x+3} \cdot \frac{2x^2+x-1}{x^2+6x+5}$

$$\frac{2x^2+7x-15}{4x^2-8x+3} \cdot \frac{2x^2+x-1}{x^2+6x+5} = \frac{(2x-3)(x+5)}{(2x-3)(2x-1)} \cdot \frac{(2x-1)(x+1)}{(x+5)(x+1)} = \frac{\cancel{(2x-3)}\cancel{(x+5)}}{\cancel{(2x-3)}\cancel{(2x-1)}} \cdot \frac{\cancel{(2x-1)}\cancel{(x+1)}}{\cancel{(x+5)}\cancel{(x+1)}} = 1$$

## DIVIDING RATIONAL EXPRESSIONS

### To Divide Two Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \quad b \neq 0, d \neq 0, \text{ and } c \neq 0$$

**Example 1** Divide.

a.  $\frac{3}{5} \div \frac{4}{5}$

b.  $\frac{2}{3} \div \frac{5}{6}$

Solution:

a.  $\frac{3}{5} \div \frac{4}{5} = \frac{3}{\cancel{5}} \cdot \frac{5^1}{4} = \frac{3 \cdot 1}{1 \cdot 4} = \frac{3}{4}$

b.  $\frac{2}{3} \div \frac{5}{6} = \frac{2}{\cancel{3}} \cdot \frac{6^2}{5} = \frac{2 \cdot 2}{1 \cdot 5} = \frac{4}{5}$

### To Divide Rational Expressions

Invert the divisor (the second fraction) and multiply.

**Example 2** Divide:  $\frac{7x^3}{z} \div \frac{5z^3}{3}$

Solution:  $\frac{7x^3}{z} \div \frac{5z^3}{3} = \frac{7x^3}{z} \cdot \frac{3}{5z^3} = \frac{21x^3}{5z^4}$

**Example 3** Divide:  $\frac{x^2-9}{x+4} \div \frac{x-3}{x+4}$

Solution:  $\frac{x^2-9}{x+4} \div \frac{x-3}{x+4} = \frac{x^2-9}{x+4} \cdot \frac{x+4}{x-3} = \frac{(x+3)(\cancel{x-3})}{\cancel{x+4}} \cdot \frac{\cancel{x+4}}{\cancel{x-3}} = x+3$

**Example 4** Divide:  $\frac{x^2+8x+15}{x^2} \div (x+3)^2$

Solution:  $(x+3)^2$  means  $\frac{(x+3)^2}{1}$ . Invert the divisor and multiply as shown below.

$$\frac{x^2+8x+15}{x^2} \div (x+3)^2 = \frac{x^2+8x+15}{x^2} \cdot \frac{1}{(x+3)^2} = \frac{(x+5)(\cancel{x+3})}{x^2} \cdot \frac{1}{(\cancel{x+3})(x+3)} = \frac{x+5}{x^2(x+3)}$$

**Example 5** Divide:  $\frac{12x^2-22x+8}{3x} \div \frac{3x^2+2x-8}{2x^2+4x}$

Solution:

$$\frac{12x^2-22x+8}{3x} \div \frac{3x^2+2x-8}{2x^2+4x} = \frac{12x^2-22x+8}{3x} \cdot \frac{2x^2+4x}{3x^2+2x-8} = \frac{2(6x^2-11x+4)}{3x} \cdot \frac{2x(x+2)}{(x+2)(3x-4)} = \frac{2(2x-1)(\cancel{3x-4})}{3\cancel{x}} \cdot \frac{2\cancel{x}(x+2)}{(\cancel{x+2})(\cancel{3x-4})} = \frac{4(2x-1)}{3}$$

## MULTIPLYING & DIVIDING RATIONAL EXPRESSIONS PRACTICE

Multiply and simplify.

1.  $\frac{5y^2}{3} \cdot \frac{9x}{10y}$

2.  $\frac{2x^3y}{3z^4} \cdot \frac{6xz^5}{10y^5}$

3.  $\frac{9y^2}{8} \cdot \frac{32x}{27y}$

4.  $\frac{2x^2y}{3z^3} \cdot \frac{12xz^4}{6y^3}$

5.  $\frac{3y^2}{5} \cdot \frac{10x}{15y}$

6.  $\frac{4x^2y}{2z^2} \cdot \frac{6xz^3}{20y^4}$

7.  $\frac{x+4}{3x+4y} \cdot \frac{9x^2-16y^2}{2x^2+3x-20}$

8.  $\frac{a^2-10a+21}{a-7} \cdot \frac{a^2+a-12}{(a-3)^2}$

9.  $\frac{x+1}{3x+y} \cdot \frac{9x^2-y^2}{2x^2+3x+1}$

10.  $\frac{a^2-6a+9}{a-3} \cdot \frac{a^2+3a-18}{(a-3)^2}$

11.  $\frac{5x^2}{3t^2} \cdot \frac{9t^8}{25x}$

12.  $\frac{7a^3}{10b^7} \cdot \frac{5b^3}{3a}$

13.  $\frac{3x-6}{5x} \cdot \frac{x^3}{5x-10}$

14.  $\frac{5t^3}{4t-8} \cdot \frac{6t-12}{10t}$

15.  $\frac{y^2-16}{2y+6} \cdot \frac{y+3}{y-4}$

16.  $\frac{m^2-n^2}{4m+4n} \cdot \frac{m+n}{m-n}$

17.  $\frac{x^2-16}{x^2} \cdot \frac{x^2-4x}{x^2-x-12}$

18.  $\frac{y^2+10y+25}{y^2-9} \cdot \frac{y^2+3y}{y+5}$

19.  $\frac{6-2t}{t^2+4t+4} \cdot \frac{t^3+2t^2}{t^8-9t^6}$

20.  $\frac{x^2-6x+9}{12-4x} \cdot \frac{x^6-9x^4}{x^3-3x^2}$

21.  $\frac{x^2-2x-35}{2x^3-3x^2} \cdot \frac{4x^3-9x}{7x-49}$

22.  $\frac{y^2-10y+9}{y^2-1} \cdot \frac{y+4}{y^2-5y-36}$

23.  $\frac{c^3+8}{c^5-4c^3} \cdot \frac{c^6-4c^5+4c^4}{c^2-2c+4}$

24.  $\frac{x^3-27}{x^4-9x^2} \cdot \frac{x^5-6x^4+9x^3}{x^2+3x+9}$

25.  $\frac{a^3-b^3}{3a^2+9ab+6b^2} \cdot \frac{a^2+2ab+b^2}{a^2-b^2}$

26.  $\frac{x^3+y^3}{x^2+2xy-3y^2} \cdot \frac{x^2-y^2}{3x^2+6xy+3y^2}$

27.  $\frac{4x^2-9y^2}{8x^3-27y^3} \cdot \frac{4x^2+6xy+9y^2}{4x^2+12xy+9y^2}$

28.  $\frac{3x^2-3y^2}{27x^3-8y^3} \cdot \frac{6x^2+5xy-6y^2}{6x^2+12xy+6y^2}$

Divide and simplify.

29.  $28p^2q^4 \div \frac{4pq^4}{5r}$

30.  $\frac{r^3s}{t} \div \frac{rs^3}{t^3}$

31.  $24e^2d^4 \div \frac{3cd^4}{5f}$

32.  $\frac{u^5x}{y} \div \frac{ux^2}{y^4}$

33.  $\frac{m^5n}{p} \div \frac{mn^3}{p^4}$

34.  $\frac{3x^2+4x+1}{3x^2-5x-2} \div \frac{x^2-2x-3}{-5x^2+25x-30}$

35.  $\frac{2x^2+5x+3}{2x^2+7x+6} \div \frac{x^2+6x+5}{-5x^2-35x-50}$

36.  $\frac{30}{y^2+4y-12} \div \frac{6y}{y-2}$

37.  $\frac{15}{y^2+2y-8} \div \frac{5y}{y-2}$

38.  $\frac{x^2+3x-28}{x^2+4x+4} \div \frac{x^2-49}{x^2-5x-14}$

39.  $\frac{16a^7}{3b^5} \div \frac{8a^3}{6b}$

40.  $\frac{9x^5}{8y^2} \div \frac{3x}{16y^9}$

41.  $\frac{3y+15}{y^7} \div \frac{y+5}{y^2}$

42.  $\frac{6x+12}{x^8} \div \frac{x+2}{x^3}$

43.  $\frac{y^2-9}{y^2} \div \frac{y^5+3y^4}{y+2}$

44.  $\frac{x^2-4}{x^3} \div \frac{x^5-2x^4}{x+4}$

45.  $\frac{4a^2-1}{a^2-4} \div \frac{2a-1}{a-2}$

46.  $\frac{25x^2-4}{x^2-9} \div \frac{5x-2}{x+3}$

47.  $\frac{x^2-y^2}{4x+4y} \div \frac{3y-3x}{12x^2}$

48.  $\frac{5y-5x}{15y^3} \div \frac{x^2-y^2}{3x+3y}$

49.  $\frac{x^2-16}{x^2-10x+25} \div \frac{3x-12}{x^2-3x-10}$

50.  $\frac{y^2-36}{y^2-8y+16} \div \frac{3y-18}{y^2-y-12}$

51.  $\frac{y^3+3y}{y^2-9} \div \frac{y^2+5y-14}{y^2+4y-21}$

52.  $\frac{a^3+4a}{a^2-16} \div \frac{a^2+8a+15}{a^2+a-20}$

53.  $\frac{x^3-64}{x^3+64} \div \frac{x^2-16}{x^2-4x+16}$

54.  $\frac{8y^3-27}{64y^3-1} \div \frac{4y^2-9}{16y^2+4y+1}$

55.  $\frac{8a^3+b^3}{2a^2+3ab+b^2} \div \frac{8a^2-4ab+2b^2}{4a^2+4ab+b^2}$

56.  $\frac{x^3+8y^3}{2x^2+5xy+2y^2} \div \frac{x^3-2x^2y+4xy^2}{8x^2-2y^2}$

### PRACTICE ANSWERS

1.  $\frac{3xy}{2}$

2.  $\frac{2x^4z}{5y^4}$

3.  $\frac{4xy}{3}$

4.  $\frac{4x^3z}{3y^2}$

5.  $\frac{2xy}{5}$

6.  $\frac{3x^3z}{5y^3}$

7.  $\frac{3x-4y}{2x-5}$

8.  $a + 4$

9.  $\frac{3x-y}{2x+1}$

10.  $a + 6$

11.  $\frac{3xt^6}{5}$

12.  $\frac{7a^2}{6b^4}$

13.  $\frac{3x^2}{25}$

14.  $\frac{3t^2}{4}$

15.  $\frac{y+4}{2}$

16.  $\frac{m+n}{4}$

17.  $\frac{(x+4)(x-4)}{x(x+3)}$

18.  $\frac{y(y+5)}{y-3}$

19.  $\frac{-2}{t^4(t+2)(t+3)}$

20.  $-\frac{x^2(x+3)(x-3)}{4}$

21.  $\frac{(x+5)(2x+3)}{7x}$

22.  $\frac{1}{y+1}$

23.  $c(c-2)$

24.  $\frac{x(x-3)^2}{x+3}$

25.  $\frac{a^2+ab+b^2}{3(a+2b)}$

26.  $\frac{x^2-xy+y^2}{3(x+3y)}$

27.  $\frac{1}{2x+3y}$

28.  $\frac{(x-y)(2x+3y)}{2(x+y)(9x^2+6xy+4y^2)}$

29.  $35pr$

30.  $\frac{r^2t^2}{s^2}$

31.  $\frac{40e^2f}{c}$

32.  $\frac{u^4y^3}{x}$

33.  $\frac{m^4p^3}{n^2}$

34.  $-5$

35.  $-5$

36.  $\frac{5}{y(y+6)}$

37.  $\frac{3}{y(y+4)}$

38.  $\frac{x-4}{x+2}$

39.  $\frac{4a^4}{b^4}$

40.  $6x^4y^7$

41.  $\frac{3}{y^5}$

42.  $\frac{6}{x^5}$

43.  $\frac{(y-3)(y+2)}{y^6}$

44.  $\frac{(x+2)(x+4)}{x^7}$

45.  $\frac{2a+1}{a+2}$

46.  $\frac{5x+2}{x-3}$

47.  $-x^2$

48.  $-\frac{1}{y^3}$

49.  $\frac{(x+4)(x+2)}{3(x-5)}$

50.  $\frac{(y+6)(y+3)}{3(y-4)}$

51.  $\frac{y(y^2+3)}{(y+3)(y-2)}$

52.  $\frac{a(a^2+4)}{(a+4)(a+3)}$

53.  $\frac{x^2+4x+16}{(x+4)^2}$

54.  $\frac{4y^2+6y+9}{(4y-1)(2y+3)}$

55.  $\frac{(2a+b)^2}{2(a+b)}$

56.  $\frac{2(2x-y)}{x}$